ĐẠI HỌC QUỐC GIA HÀ NỘI
TRƯỜNG ĐẠI HỌC KHOA HỌC TỰ NHIÊN

ĐỀ THI KẾT THÚC HỌC PHẦN
HỌC KỲ II, NĂM HỌC 2021-2022

Tên học phần: Quantum mechanics

Mã học phần: PHY2306 Số tín chỉ: 04 Đề số: 01

Dành cho sinh viên lớp học phần: PHY2306E QTL

Thời gian làm bài: 90 phút (không kể thời gian phát đề)

General instruction:

(1) Students are not allowed to use any documents.

(2) Students may need following relations:

(i) For a 1D harmonic oscillator, the relations between annihilation, creation, coordinate and momentum operators:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}^\dagger + \hat{a}\right), \quad \text{and} \quad \hat{p} = i\sqrt{\frac{m\omega\hbar}{2}}\left(\hat{a}^\dagger - \hat{a}\right)$$

(ii) For a particle in central potential, the relations between annihilation, creation and angular momentum operators:

$$\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$$

$$\hat{\mathbf{J}}^2 = \frac{1}{2}(\hat{J}_+\hat{J}_- + \hat{J}_-\hat{J}_+) + \hat{J}_z^2$$

(iii) Pauli matrices:

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Part 1. For all students (9/10 points)

Instruction for this part: All students have to answers all questions in this part

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Question 1. (5/10 points)

(a) Solve time-independent Schrodinger equation for a 1D particle of mass m confined in an infinite square well of size L to find stationary states of the particle and its energy. Normalize these states.

(b) Assuming that initially the particle is in the ground state in this potential well. Schetch the wave function of the particle.

(c) We suddently double the size of the well by moving the right wall of the well to the right. In the figure of (b), schetch the doubled-size potential well. What is the posibility to find the particle in the ground state of the new potential well?

Question 2. (2/10 points)
Derive the matrix form of annihilation, creation, momentum and coordinate operators of a 1D harmonic oscillator.

Question 3. (2/10 points)
Given that a particle in a central potential has its angular momentum quantized in magnitude and orientation. The corresponding quantum number is j and m so that a quantum state is denoted by $|j,m\rangle$. $\hat{J}_x, \hat{J}_y, \hat{J}_z$ are three components of angular momentum operator. Calculate $\langle J_x^2\rangle$; $\langle J_y^2\rangle$; $\langle J_z^2\rangle$; $\langle J^2\rangle$ of the particle if it is in quantum state $|j,m\rangle$.

Part 2. Optional questions (1/10 points)
Instruction for this part: Students have to answer one of two questions in this part

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Question 4(O1). (1/10 points)
Find eigenvalues and eigenvectors of Pauli matrices.

Question 4(O2). (1/10 points)
Write Planck-Eistein relations for electromagnetic radiation and de Broglie relations for matters and state their significances to quantum mechanics.

THE END