MAT2411E 1-2, Optimization, Spring 2022, Final Examination

Version 1, Time: 90 minutes, Number of credits: 3

Problem 1. (2.5 points) Solve the following problem by the two–phase simplex method.

$$f(x) = 5x_1 + 3x_2 - x_3 \to \min$$

$$\begin{cases} x_1 + x_2 + x_3 \le 7 \\ 2x_1 + 3x_2 - x_3 = 2 \\ x_1 + 3x_2 - x_3 = 0 \\ x_j \ge 0, j = 1,2,3. \end{cases}$$

Problem 2. (2.5 points) Solve the following problem by the dual simplex method.

$$f(x) = x_1 + 4x_2 - 2x_3 - 3x_4 \to \min$$

$$\begin{cases} 2x_1 + x_2 - 3x_3 + x_4 = 12 \\ x_1 - x_2 + 2x_3 - x_4 = 4 \\ x_1 + x_2 + x_3 + x_4 = 14 \\ x_j \ge 0, j = 1,\dots,4. \end{cases}$$

Problem 3. (2.5 points) Given the linear programming problem

$$f(x) = x_1 - x_3 + 3x_4 \to \max$$

$$\begin{cases} 2x_1 - x_2 + 3x_3 - 5x_4 \ge 0 \\ 3x_1 - 4x_3 + 6x_4 \le 5 \\ 2x_1 - x_2 + 4x_3 - 5x_4 \le 1 \\ x_j \ge 0, j = 1,\dots,4. \end{cases}$$

a. Prove that $\overline{x} = (1,0,1,1)$ is an optimal solution. (1 point)

b. Write the dual problem. (1 point)

c. Find an optimal solution to the dual problem. (0.5 point)

Problem 4. (2.5 points) Given the function

$$f(x,y) = (x^2 + y^2)e^{-(x^2+y^2)}.$$

a. Find all stationary points of this function. (0.5 point)

b. Show that f attains its global minimum value at $(0,0)$. (0.5 point)

c. Show that f attains its global maximum value at all points (x,y) on the unit circle $x^2 + y^2 = 1$. (0.5 point)

d. Use the gradient descent method with backtracking line search, given the starting point $x^{(0)} = (0.5, 0.5)$, step length $t_0 = 1$, and parameters $\alpha = 0.2$, $\beta = 0.5$ to find the iterate $x^{(1)}$ of the optimization problem $f(x,y) \to \min$. (1 point)

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[0] Sinh viên được sử dụng máy tính cầm tay và từ điển Anh - Việt.