

VIETNAM NATIONAL UNIVERSITY, HANOI  
 University of Engineering and Technology

Date: December 24, 2014

**FINAL EXAMINATION**  
 Course: **Signals and Systems (ELT2035 4)**  
 Duration: 90 minutes

**Part 1 (Multiple-choice questions):** For problems in this part, you only have to give the letter of the correct answer (A/B/C/D). Explanations are not required.

**Problem 1.** Which one of the following signals is an energy signal?

- A.  $x(t) = \sin(3\pi t)[u(t) - 2u(t-4)]$
- B.  $x(n) = 2^{-|n|} \cos(\pi n/3)$
- C.  $x(n) = nu(-n)$
- D.  $x(t) = (e^{2t} - e^{-3t})u(t)$

Answer: B

**Problem 2.** Which one of the following LTI systems can be both causal and stable?

- A.  $y(t) - \frac{dy(t)}{dt} + \frac{d^2 y(t)}{dt^2} = x(t) + \frac{dx(t)}{dt}$
- B.  $y(n) + 2y(n-1) = x(n)$
- C.  $\frac{dy(t)}{dt} + \frac{d^2 y(t)}{dt^2} = 2x(t)$
- D.  $8y(n) + 2y(n-1) - y(n-2) = x(n-1)$

Answer: D

**Problem 3.** The frequency response of a continuous-time LTI system exists and is given by:

$$H(\omega) = \frac{2}{\omega^2 + 3j\omega - 2}$$

which one of the following statements about this system is correct?

- A. This system is causal.
- B. This system is anti-causal.
- C. This system is non-causal (not causal nor anti-causal).

D. This system is not stable.

*Answer: B*

**Problem 4.** Which one of the following statements is correct?

- A. The Fourier spectrum of a discrete-time energy signal is continuous and periodic.
- B. The Fourier spectrum of a discrete-time energy signal is continuous and non-periodic.
- C. The Fourier spectrum of a discrete-time energy signal is discrete and periodic.
- D. The Fourier spectrum of a discrete-time energy signal is discrete and non-periodic.

*Answer: A*

**Part 2 (Exercises):** For problems in this part, detailed explanations/derivations that lead to the answer must be provided.

**Problem 5.** Given a causal LTI system described by the following differential equation:

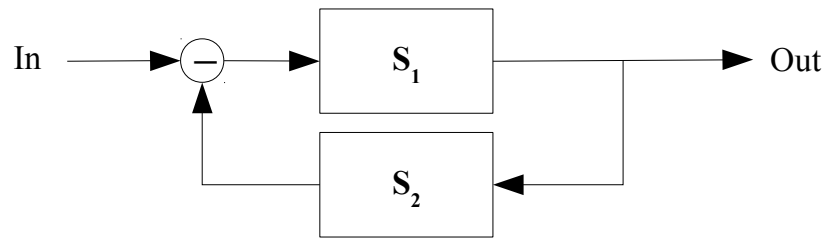
$$y(t) + 3 \frac{dy(t)}{dt} + 2 \frac{d^2 y(t)}{dt^2} = x(t) + 2 \frac{dx(t)}{dt}$$

- a) Determine the impulse response of the given system.
- b) Determine the initial response  $y_0(t)$  of the given system to the following initial conditions:  $y(0^-) = -1$  and  $\left. \frac{dy(t)}{dt} \right|_{t=0^-} = -1$ .
- c) Determine the zero-state response  $y_s(t)$  of the given system to the input signal  $x(t) = e^{-2t} u(t)$ .

*Answers:*

- a) Inverse Laplace transform of  $H(s) = \frac{2s+1}{2s^2+3s+1} = \frac{1}{s+1}$  ( $h(t)$  is causal).
- b) Use unilateral Laplace transform or solve the homogeneous equation with initial conditions directly.
- c) Inverse Laplace transform of  $Y_s(s) = H(s)X(s)$

**Problem 6.** Given a system **T** described by the following block diagram:



in which,  $S_1$  is a continuous-time linear time-invariant system described by the differential equation  $y(t) + \frac{dy(t)}{dt} = \frac{dx(t)}{dt}$  and the feedback block  $S_2$  has the transfer function of  $H_2(s) = \frac{1}{s-1}$ .

- Determine the transfer function of **T**.
- Determine the frequency response of system **T** when: i) **T** is causal, and ii) **T** is anti-causal.
- Determine the output of system **T** to the input  $x(t) = \sin(t/3)$  when: i) **T** is causal, and ii) **T** is anti-causal.

*Answers:*

- $H(s) = \frac{s(s-1)}{s^2+s-1}$
- i)  $H(\omega) = H(s)_{s=j\omega}$ , because the system is stable; ii) not exist, because the system is not stable.
- i)  $y(\omega) = \frac{1}{2j} H(j/3) e^{t/3} - \frac{1}{2j} H(-j/3) e^{-t/3}$ ; ii) infinity, because the frequency response does not converge at the frequency of the input sinusoidal signal.

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