VIETNAM NATIONAL UNIVERSITY, HANOI
University of Engineering and Technology

Date: June 17, 2016

FINAL EXAMINATION - ANSWERS
Course: **Signals and Systems (ELT2035)**
Duration: 90 minutes

Part 1 (Multiple-choice questions)*: For problems in this part, you only have to give the letter of the correct answer (A/B/C/D). Explanations are not required.*

Problem 1. (1 point) Which one of the systems described by the following input-output relations is a stable linear time-invariant system?

A. $y(t)=2x(t)\sin(3\pi t)$

B. $y(n)-y(n-1)=2x(n)$

C. $y(t)=2^{x(t)}u(t-1)$

D. $y(n)=2x(n)+x(n-1)$

Answer: D

Problem 2. (1 point) A continuous-time linear time-invariant system is described by the following transfer function:

$$H(s)=\frac{2s-1}{s^2+s-2}$$

Among the following statements about the given system, which one is TRUE?

A. The system can be both causal and stable.

B. The system can be both anti-causal and stable.

C. If the system is causal, then it is not stable.

D. If the system is stable, then it is neither causal nor anti-causal.

Answer: D

Problem 3. (1 point) Which one of the following signals is NOT an energy signal?

A. $x(t)=e^{-2t+1}u(t-1)$

B. $x(n)=2^{-|n|}$

C. $x(t)=[\cos(\pi t/2+\pi/4)]^{-1}[u(t)-u(t-10)]$

D. $x(n)=[\cos(\pi n/2+\pi/4)]^{-1}[u(n)-u(n-10)]$

Answer: C

Problem 4. Given the following discrete-time periodic signal:

$$x(n)=e^{j\pi n/2}+\cos(\pi n/3+\pi/4)+2\sin(\pi n/4)+1$$

What is the fundamental period of the given signal?

A. $T_0=6$ (samples)

B. $T_0=12$ (samples)

C. $T_0=18$ (samples)

D. $T_0=24$ (samples)

Answer: D

<u>Part 2 (Exercises)</u>*:For problems in this part, detailed explanations/derivations that lead to the answer must be provided.*

Problem 5. (3 points) Given a continuous-time causal linear time-invariant system described by the following differential equation:

$$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+\frac{y(t)}{2}=2\frac{dx(t)}{dt}+x(t)$$

a) Is the given system stable or not?

Answer: Stable, because all system roots lie in the left half of the s-plane.

b) Determine the system impulse response.

Answer:

$$H(s)=\frac{2s+1}{\left(s+\frac{1-j}{2}\right)\left(s+\frac{1+j}{2}\right)}=\frac{1}{s+\frac{1-j}{2}}+\frac{1}{s+\frac{1+j}{2}}$$

$$h(t)=(e^{-\frac{1-j}{2}t}+e^{-\frac{1+j}{2}t})u(t)$$

c) Determine the system response to the input $x(t)=e^{-t/2}u(t)$.

Answer:

$$X(s)=\frac{1}{s+1/2}$$

$$Y(s)=\frac{2s+1}{\left(s+\frac{1-j}{2}\right)\left(s+\frac{1+j}{2}\right)}\frac{1}{s+1/2}=\frac{2}{\left(s+\frac{1-j}{2}\right)\left(s+\frac{1+j}{2}\right)}$$

$$y(t)=2[-je^{-\frac{1-j}{2}t}+je^{-\frac{1+j}{2}t}]u(t)$$

Problem 6. (3 points) Given a discrete-time linear time-invariant system having the impulse response $h(n)=2^{-n}u(n-1)$.

a) Determine the system frequency response.

Answer:

$$H(\Omega)=\frac{e^{-j\Omega}}{2-e^{-j\Omega}}$$

b) Determine the system response to the input signal $x(n)=\sin(\pi n/2+\pi/3)+2\cos(\pi n)+3$.

Answer:

$$y(n)=\frac{1}{2j}H(\pi/2)e^{j(\pi n/2+\pi/3)}-\frac{1}{2j}H(-\pi/2)e^{-j(\pi n/2+\pi/3)}+H(\pi)e^{j\pi n}+H(-\pi)e^{-j\pi n}+3H(0)$$

c) Determine the system response to the input signal $x(n)=3^n[u(n)-u(n-10)]$.

Answer:

$$y(n)=x(n)*h(n)=\sum_{k=0}^{9}3^k 2^{-(n-k)}u(n-k-1)$$

If $n<10$ *then* $y(n)=\sum_{k=0}^{n-1}3^k 2^{-(n-k)}$ ...

If $n>=10$ *then* $y(n)=\sum_{k=0}^{9}3^k 2^{-(n-k)}$ ...

***** END *****