smartosc

$\boxed{2.4}$

a. Không gian mẫu:

$S = \{(-2;-2); (-2;-1); (-2;0); (2;0); (2;1); (2;2)\}$

b. Biến cố: Tín hiệu truyền chắc chắn là (2)

$E = \{(2;1); (2;2)\}$

c. $Y = 0$: Tín hiệu truyền không thể xác định dựa vào tín hiệu bên nhận được

$\boxed{2.19}$

$S = \{-1; 0; 1\}$

a. $\emptyset$, $\{-1\}$; $\{0\}$; $\{-1\}$; $\{-1;0\}$; $\{-1;1\}$; $\{0;1\}$; $\{-1;0;1\}$

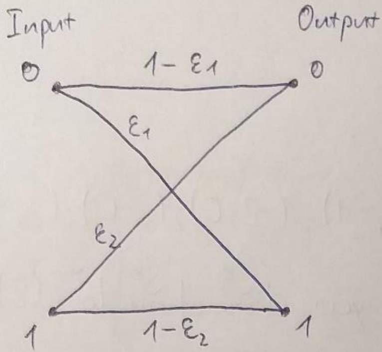
8 tập con

b. Kết quả của thực nghiệm bao gồm các cặp không có 2 số nào giống nhau

$S = \{(-1;0); (-1;1); (0;1); (0;-1); (1;-1);$
$(1;0)\}$

Powerset $S' = 2^6 = 64$

$\boxed{2.77}$



a. Tính xác suất mà output là 0:

$$P[Y=0] = P[Y=0|X=1].P[X=1] + P[Y=0|X=0].P[X=0]$$

$$= \varepsilon_2(1-p) + (1-\varepsilon_1).p$$

b. $$P[X=0|Y=1] = \frac{P[Y=1|X=0].P[X=0]}{P[Y=1]}$$

$$= \frac{\varepsilon_1.p}{P[Y=1|X=0].P[X=0] + P[Y=1|X=1].P[X=1]}$$

$$= \frac{\varepsilon_1.p}{\varepsilon_1.p + (1-\varepsilon_2).(1-p)}$$

$$P[X=1|Y=1] = \frac{(1-\varepsilon_2)(1-p)}{\varepsilon_1 p + (1-\varepsilon_2)(1-p)}$$

$\boxed{2.1}$

a. $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

b. $A = \{1, 2, 3, 4\}$

$B = \{2, 3, 4, 5, 6, 7, 8\}$

$D = \{1, 3, 5, 7, 9, 11\}$

c. $A \cap B \cap D = \{3\}$

$\bar{A} \cap B = \{5, 6, 7, 8\}$

$A \cup (B \cap D^c) = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B) \cap D^c = \{2, 4, 6, 8\}$

$\boxed{2.2}$

a.

x \ y	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

b.

x \ y	1	2	3	4	5	6
1	✓					
2	✓	✓				
3	✓	✓	✓			
4	✓	✓	✓	✓		
5	✓	✓	✓	✓	✓	
6	✓	✓	✓	✓	✓	✓

$A = \{N_1 < N_2\}^c = \{N_1 \geq N_2\}$

c.

x \ y	1	2	3	4	5	6
1						
2						
3						
4						
5						
6	✓	✓	✓	✓	✓	✓

$B = \{N_1 = 6\}$

d. B là tập con của A nên khi B xảy ra thì A cũng xảy ra => B kéo theo A

$\boxed{2.15}$

a. $D = A_1 \cap A_2 \cap A_3$

b. $D = A_1 \cup A_2 \cup A_3$

c. $D = (A_1 \cap A_2 \cap A_3) \cup (A_1^c \cap A_2 \cap A_3) \cup$
$(A_1 \cap A_2^c \cap A_3) \cup (A_1 \cap A_2 \cap A_3^c)$

$\boxed{2.36}$

Gọi x là thời gian tồn tại:

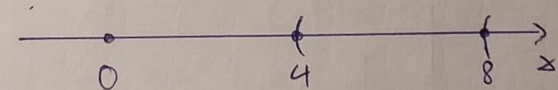
$A = \{x > 4\}$

$B = \{x > 8\}$

a. $P[A \cap B] = P[\{x > 4\} \cap \{x > 8\}] = P[\{x > 8\}] = \frac{1}{8}$

$P[A \cup B] = P[\{x > 4\} \cup \{x > 8\}] = P[\{x > 4\}] = \frac{1}{4}$

b.



$P[\{x > 6\}] = P[\{6 < x \leq 12\} \cup \{x > 12\}]$

$= P[\{6 < x \leq 12\}] + P[\{x > 12\}]$

$\Rightarrow P[\{6 < x \leq 12\}] = P[\{x > 6\}] - P[\{x > 12\}]$

$= \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$

$\boxed{3.12}$

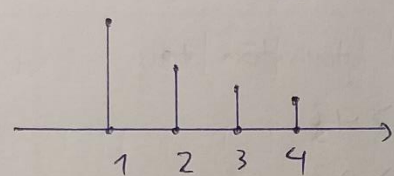
a. $P_k = P_1/k$

$$1 = P_1 + P_2 + P_3 + P_4 = P_1\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \frac{25}{12} P_1$$

$$\Rightarrow P_1 = \frac{12}{25} \quad \Rightarrow P_2 = \frac{6}{25}$$

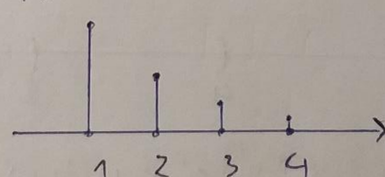
$$P_3 = \frac{4}{25}$$

$$P_4 = \frac{3}{25}$$



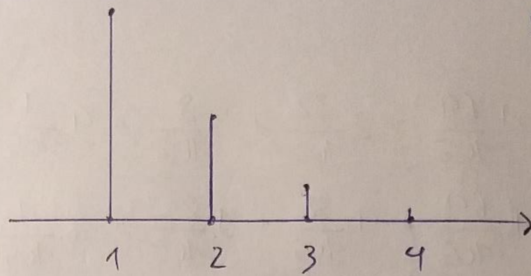
b. $$1 = P_1\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = \frac{15}{8} P_1$$

$$\Rightarrow P_1 = \frac{8}{15}; \quad P_2 = \frac{4}{15}; \; P_3 = \frac{2}{15}; \; P_4 = \frac{1}{15}$$



c. $$1 = P_1\left(1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{64}\right) = \frac{105}{64} P_1$$

$$\Rightarrow P_1 = \frac{64}{105} \Rightarrow \quad P_2 = \frac{32}{105}; \; P_3 = \frac{8}{105}; \; P_4 = \frac{1}{105}$$



$\boxed{3.13}$

a. $1 = \sum_{k=1}^{\infty} \frac{c}{k^2} = c \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} = 1,6449 \Rightarrow c = 0,608$

b. $P[X > 4] = 1 - P[X \leq 3] = 1 - c\left[1 + \frac{1}{4} + \frac{1}{9}\right] = 0,1675$

c. $P[6 \leq X \leq 8] = c\left[\frac{1}{36} + \frac{1}{49} + \frac{1}{64}\right] = 0,390$

$\boxed{3.22}$

a. $E[X] = 1.\frac{12}{25} + 2\frac{6}{25} + 3.\frac{4}{25} + 4\frac{3}{25} = \frac{48}{25} = 1,92$

$E[X^2] = 1.\frac{12}{25} + 4.\frac{6}{25} + 9.\frac{4}{25} + 16.\frac{3}{25} = \frac{120}{25} =$

$VAR[X] = \frac{120}{25} - \left(\frac{48}{25}\right)^2 = \frac{696}{625} = 1,114$

b. $E[X] = 1.\frac{8}{15} + 2.\frac{4}{15} + 3.\frac{2}{15} + 4.\frac{1}{15} = \frac{26}{15} = 1,73$

$E[X^2] = 1.\frac{8}{15} + 4.\frac{4}{15} + 9.\frac{2}{15} + 16.\frac{1}{15} = \underline{58}$

$VAR[X] = \frac{58}{15} - \left(\frac{26}{15}\right)^2 = \frac{194}{225} = 0,862$

c. $E[X] = 1.\frac{64}{105} + 2.\frac{32}{105} + 3.\frac{8}{105} + 4.\frac{1}{105} = \frac{156}{105} = 1,48$

$E[X^2] = 1.\frac{64}{105} + 4.\frac{32}{105} + 9.\frac{8}{105} + 16.\frac{1}{105} = \frac{280}{105}$

$VAR[X] = \frac{280}{105} - \left(\frac{156}{105}\right)^2 = \frac{5064}{(105)^2} = 0,459$

$\boxed{3.31}$

a. $P[X=k] = \binom{n}{j}\left(\frac{1}{2}\right)^n$

$E[aX^2 + bX] = aE[X^2] + bE[X]$

$E[X] = \sum_{j=0}^{n} j\binom{n}{j}\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n \sum_{j=0}^{n} j\frac{n!}{j!(n-j)!}$

$= \left(\frac{1}{2}\right)^n \sum_{j=1}^{n} \frac{n!}{(j-1)!(n-j)!}$

Đặt $j' = j-1 \Rightarrow E[X] = \left(\frac{1}{2}\right)^n n \sum_{j'=0}^{n-1} \frac{(n-1)!}{j'!(n-1-j')!}$

$= n\left(\frac{1}{2}\right)^n \sum_{j'=0}^{n-1} \binom{n-1}{j'}$

$= n\left(\frac{1}{2}\right)^n . 2^{n-1} = \frac{n}{2}$

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$$E[X^2] = \sum_{j=0}^{n} j^2 \binom{n}{j} \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n n \sum_{j=1}^{n} j \frac{(n-1)!}{(j-1)!(n-j)!}$$

$$= n\left(\frac{1}{2}\right)^n \sum_{j'=0}^{n-1} (j'+1)\binom{n-1}{j'}$$

$$= n\left(\frac{1}{2}\right)\left[\sum_{j'=0}^{n-1} j'\binom{n-1}{j'}\left(\frac{1}{2}\right)^{n-1} + \sum_{j'=0}^{n-1}\binom{n-1}{j'}\left(\frac{1}{2}\right)^{n-1}\right]$$

$$= \frac{n}{2}\left[\frac{n}{2}+1\right]$$

$$E[aX^2+bX] = a\frac{n}{2}\left(\frac{n}{2}+1\right) + b\frac{n}{2}$$

b. $$E[a^X] = \sum_{j=0}^{n} a^j \binom{n}{j}\left(\frac{1}{2}\right)^j = \sum_{j=0}^{n}\binom{n}{j}\left(\frac{a}{2}\right)^j = \left(1+\frac{a}{2}\right)^n$$

$\boxed{3.33}$

a. $$E[(X-10)^+] = \sum_{i=1}^{15}(i-10)P[X=i] = P_1\sum_{i=1}^{15}(i-10)\frac{1}{i}$$

$$= 0,33373$$

b. $$E[(X-10)^+] = P_1\sum_{i=1}^{15}(i-10)2^{-(i-1)} = 0,00174$$

c. $$E[(X-10)^+] = P_1\sum_{i=1}^{15}(i-10)2^{-i(i-1)/2} = 1,69.10^{-17}$$

$\boxed{3.50}$

a. $$\frac{P_k}{P_{k-1}} = \frac{\binom{n}{k} p^k q^{n-k}}{\binom{n}{k-1} p^{k-1} q^{n-k+1}} =$$

$$= \frac{\frac{n!}{k!(n-k)!}}{\frac{n!}{(k-1)!}} \cdot \frac{p}{q} = \frac{(n-k+1)p}{kq}$$

$$= \frac{(n+1)p - k(1-q)}{kq} = 1 + \frac{(n+1)p - k}{kq}$$

b. Giả sử $(n+1)p$ không phải số nguyên:

với: $0 \le k \le [(n+1)p] < (n+1)p$

$$(n+1)p - k > 0$$

$$\Rightarrow \frac{P_k}{P_{k-1}} = 1 + \frac{(n+1)p-k}{kq} > 1$$

$\Rightarrow P_k$ tăng, k tăng từ 0 đến $[(n+1)p]$

với: $k > (n+1)p \ge [(n+1)p]$

$$(n+1)p - k < 0$$

$$\Rightarrow \frac{P_k}{P_{k-1}} = 1 + \frac{(n+1)p-k}{kq} < 1$$

$\Rightarrow P_k$ giảm, k tăng qua $[(n+1)p]$

P_k lớn nhất khi $k_{max} = [(n+1)p]$

$$\frac{P_{k_{max}}}{P_{k_{max}-1}} = 1 \Rightarrow P_{k_{max}} = P_{k_{max}-1}$$

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$\boxed{3.52}$

a. $P[N=k] = (1-p)^k p$ với $k = \{0, 1, 2, \dots\}$

b. $E[N] = \sum_{k=0}^{\infty} k(1-p)^k p = (1-p)\, p \sum_{k=0}^{\infty} k(1-p)^{k-1}$

$= (1-p)\, p \cdot \frac{1}{(1-(1-p))^2} = \frac{1-p}{p}$

$\boxed{3.55}$

$\boxed{3.62}$

$$\frac{P_k}{P_{k-1}} = \frac{\frac{\alpha^k}{k!}e^{-\alpha}}{\frac{\alpha^{k-1}}{(k-1)!}e^{-\alpha}} = \frac{\alpha}{k}$$

Nếu $\alpha < 1$ thì $\frac{P_k}{P_{k-1}} = \frac{\alpha}{k} < 1$ với $k \geq 1$

P_k giảm khi k tăng từ 0

P_k ~~đạt~~ lớn nhất khi $k = 0$

Nếu $\alpha > 1$ thì

với $0 \leq k \leq [\alpha] < \alpha$, $\frac{P_k}{P_{k-1}} = \frac{\alpha}{k} > 1$

$\Rightarrow P_k$ tăng từ $k = 0$ đến $k = [\alpha]$

với $[\alpha] < \alpha < k$, $\frac{P_k}{P_{k-1}} = \frac{\alpha}{k} < 1$

$\Rightarrow P_k$ giảm khi k tăng qua $[\alpha]$

P_k lớn nhất khi $k_{max} = [\alpha]$

Nếu $\alpha = [\alpha]$ thì với $k = [\alpha]$: $\frac{P_k}{P_{k-1}} = 1 \Rightarrow P_{k_{max}} = P_{k_{max}-1}$

$\boxed{3.67}$

$p = 10^{-6} \qquad n = 10^{4} \qquad np = 10^{-2}$

a. $P[N=0] = e^{-np} = 0{,}99$

$P[N \leq 3] = \sum_{k=0}^{3} \frac{(0{,}01)^k}{k!} e^{-np} \approx 1$

b. $0{,}99 = P[N \geqslant 1] = 1 - P[N=0] = 1 - e^{-np}$

$\hookrightarrow 0{,}01$

$\Rightarrow p = \frac{\ln 100}{n} = 4{,}6.10^{-6}$

$\boxed{3.70}$

$P_k = \frac{1}{c_{10}} \frac{1}{k} \qquad k = 1, \dots, 10$

$c_{10} = 2{,}93$

$P_1 = \frac{1}{2{,}93} = 0{,}34$

$P[X > 5] = \frac{1}{c_{10}} \left[\frac{1}{6} + \dots + \frac{1}{10} \right] = 0{,}2204$

$\boxed{3.72}$

$P_k = \frac{1}{c_L} \frac{1}{k}$

$\ln P_k = \ln \frac{1}{c_L} + \ln \frac{1}{k} = -\ln k - \ln c_L$

3.74

$$P_k = \frac{1}{c_L}\,\frac{1}{k} \qquad L = 10^4$$

$$c_L = \ln 10^4 + 0{,}57721 = 9{,}7876$$

$$0{,}99 = P[X \le k_0] = \frac{1}{c_{10000}} \sum_{j=1}^{k_0} \frac{1}{j} = \frac{c_{k_0}}{c_{10000}} \approx \frac{\ln k_0 + 0{,}57721}{9{,}7876}$$

$$\ln k_0 \approx 0{,}99(9{,}7876) - 0{,}57721 = 9087$$

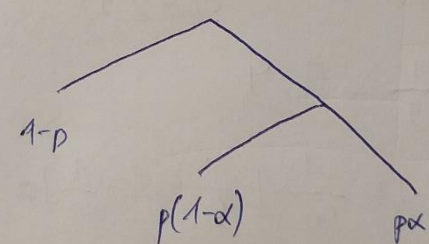
3.88

a. $P[\text{pass}] = (1-p) + p(1-\alpha)$

$P[\text{fail}] = p\alpha$

$P[k] = [(1-p) + p(1-\alpha)]^{k-1} (p\alpha)^{-1}$

b.



3.89

$$\sum_{k=1}^{\infty} k p^{k-1}(1-p) = (1-p)\sum_{k=1}^{\infty} \frac{dp^k}{dp} = (1-p)\frac{d}{dp}\sum_{k=1}^{\infty} p^k$$

$$= (1-p)\frac{d}{dp}\frac{1}{1-p} = \frac{1}{1-p}$$

Thời gian truyền tin trung bình mất $\frac{2T}{1-p}$ giây

$\Rightarrow$ max rate $= \frac{1-P}{2T}$

$\boxed{3.91}$

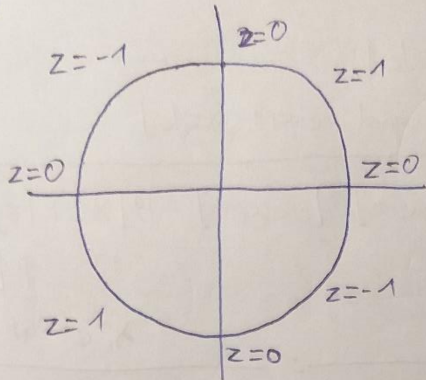
$$P[\text{signal present} \mid X = k]$$

$$= \frac{P[\text{signal present}, X = k]}{P[X = k \mid \text{signal present}]\,P[\text{present}] + P[X = k \mid \text{signal absent}]\,P[\text{absent}]}$$

$$= \frac{\frac{\lambda_1^k}{k!} e^{-\lambda_1 p}}{\frac{\lambda_1^k}{k!} e^{-\lambda_1 p} + \frac{\lambda_0^k}{k!} e^{-\lambda_0 (1-p)}} = \frac{\lambda_1^k e^{-\lambda_1 p}}{\lambda_1^k e^{-\lambda_1 p} + \lambda_0^k e^{-\lambda(1-p)}}$$

Tương tự: $P[\text{signal absent} \mid X = k] = \dfrac{\lambda_0^k e^{-\lambda_0(1-p)}}{\lambda_1^k e^{-\lambda_1 p} + \lambda_0^k e^{-\lambda_0(1-p)}}$

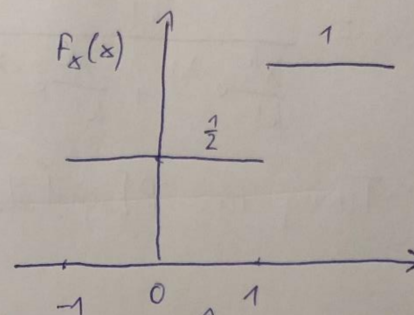
$\boxed{4.3}$

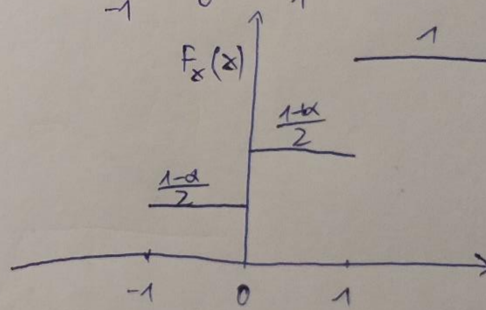


$S_X = \{-1, 0, 1\}$

$P = \left(\frac{1}{4}+\frac{1}{4}, 0, \frac{1}{4}+\frac{1}{4}\right)$

$= \left(\frac{1}{2}, 0, \frac{1}{2}\right)$

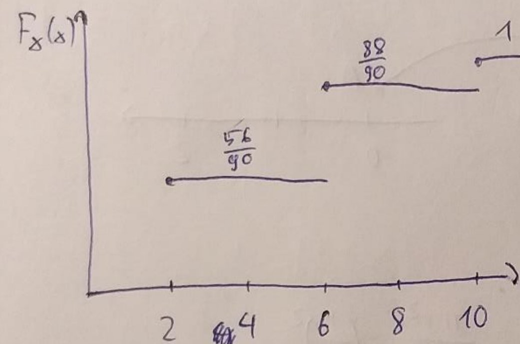




$\boxed{4.4}$

Không thay: $1+1$, $1+5$, $5+5$ $\quad S_X = \{2,6,10\}$

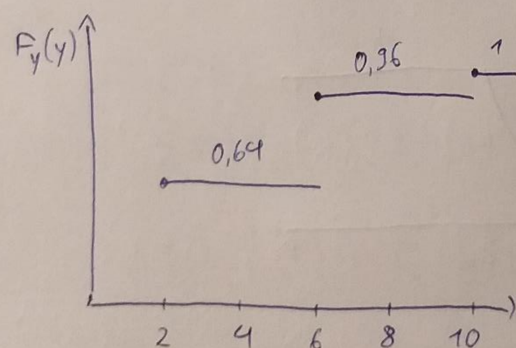
Có thay: $1+1$, $1+5$, $5+5$ $\quad S_Y = \{2,6,10\}$



$P[2] = \frac{8}{10} \cdot \frac{7}{9} = \frac{56}{90}$

$P[6] = 2 \cdot \frac{8}{10} \cdot \frac{2}{9} = \frac{32}{90}$

$P[10] = \frac{2}{10} \cdot \frac{1}{9} = \frac{2}{90}$



$P[2] = 0,8^2 = 0,64$

$P[6] = 2 . 0,8 . 0,2 = 0,32$

$P[10] = (0,2)^2 = 0,4$

$P[X=2] = \frac{56}{90}$ $\qquad P[Y=2] = \frac{64}{100}$

$P[X<7] = \frac{88}{90}$ $\qquad P[Y<7] = \frac{96}{100}$

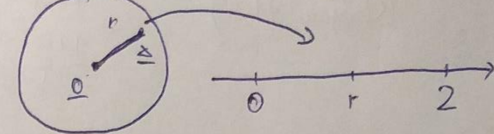
$P[X \geq 6] = 1 - \frac{56}{90} = \frac{34}{90}$ $\qquad P[Y \geq 6] = 1 - 0,64 = \frac{36}{100}$

$\boxed{4.6}$

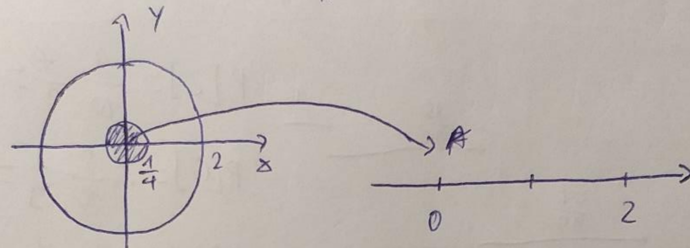
$S = \{(x,y): x^2+y^2 \leq 4\}$

$S_R = \{r: 0 \leq r \leq 2\}$

$R = \sqrt{x^2+y^2}$

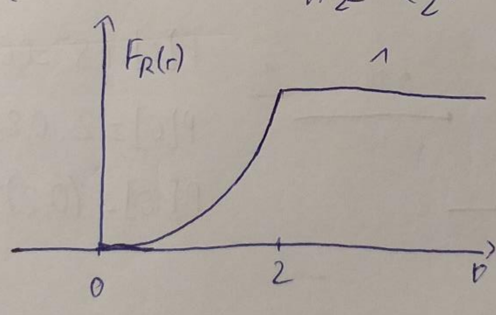


$P[A] = P[R \leq \frac{1}{4}] = \frac{\pi(\frac{1}{4})^2}{\pi\, 2^2} = \frac{1}{64}$



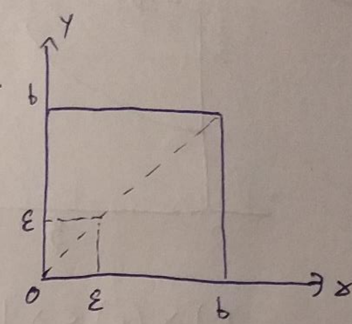
Với $0 \leq r \leq 2$

$F_R(r) = P[R \leq r] = \frac{\pi r^2}{\pi 2^2} = \left(\frac{r}{2}\right)^2$



$\boxed{4.7}$

a. $S = \{(x,y) : 0 \leq x < b,\ 0 \leq y < b\}$

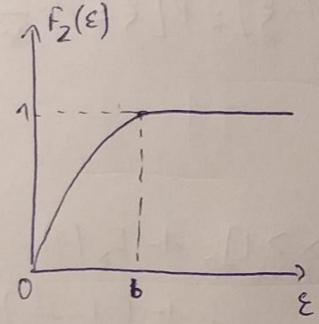


b. $S_Z = \{\varepsilon : 0 \leq \varepsilon \leq b\}$

$\min(x,y) : \varepsilon$

c. $P[Z \leq \varepsilon] = 1 - \left(\frac{b-\varepsilon}{b}\right)^2$

d.



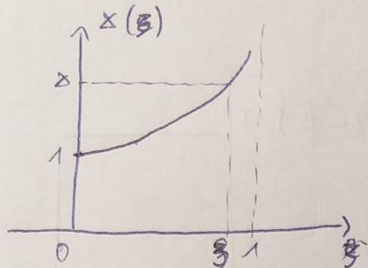
e. $P[Z > 0] = 1$

$P[Z > b] = 1$

$P[Z \leq \frac{b}{2}] = F_Z(\frac{b}{2}) = 1 - \left(\frac{b - \frac{b}{2}}{b}\right)^2 = \frac{3}{4}$

$P[Z > \frac{b}{4}] = 1 - F_Z(\frac{b}{4}) = 1 - \left(\frac{b - \frac{b}{4}}{b}\right)^2 = \frac{9}{16}$

$\boxed{4.8}$



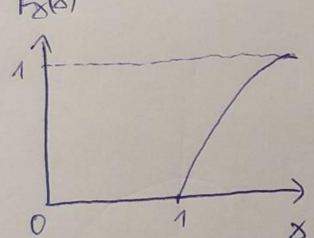
$$x(\xi) = \frac{1}{\sqrt{1-\xi}}$$

$$S_X = \{x : 1 \leq x \leq \infty\}$$

$$P[X(\xi) \leq x] = P\left[\frac{1}{\sqrt{1-\xi}} \leq x\right] = P\left[\frac{1}{1-\xi} \leq x^2\right]$$

$$= P\left[\frac{1}{x^2} \leq 1-\xi\right] = P\left[\xi \leq 1 - \frac{1}{x^2}\right]$$

$$= 1 - \frac{1}{x^2}$$



$$P[X > 1] = 1 - F_X(1)$$
$$= 1 - \left(1 - \frac{1}{1^2}\right) = 1$$

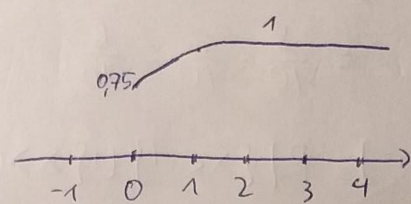
$$P[5 < X < 7] = F_X(7) - F_X(5) = \left(1 - \frac{1}{49}\right) - \left(1 - \frac{1}{25}\right)$$

$$= \frac{1}{25} - \frac{1}{49} = 0{,}01959$$

$$P[X \leq 20] = 1 - \frac{1}{400} = 0{,}9975$$

$\boxed{4.13}$

a.



Biến ngẫu nhiên kết hợp

b. $P[x \le 2] = 1 - \frac{1}{4} e^{\frac{-2}{2}} = 0,9954$

$P[x = 0] = 1 - \frac{1}{4} e^{-2(0)} = 0,75$

$P[x < 0] = 0$

$P[2 < x < 6] = P[x \le 6] - P[x \le 2]$

$= 1 - \frac{1}{4} e^{-2(6)} - 1 + \frac{1}{4} e^{-2(2)} = 0,0046$

$P[x > 10] = 1 - P[x \le 10] = 1 - (1 - \frac{1}{4} e^{-2(10)})$

$= 5.15 . 10^{-10}$

$\boxed{4.14}$

$P[x < -1] = 0$

$P[x \le -1] = \frac{2}{10}$

$P[-1 < x < -0,75] = P[x \le -0,75] - P[x \le -1]$

$= \frac{2}{10} - \frac{2}{10} = 0$

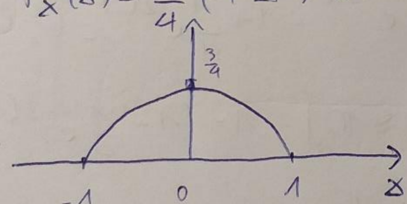
$$P[-0,5 \le O \le 0,5] = P[X \le 0,5] - P[X \le -0,5]$$
$$= \frac{8}{10} - \frac{2}{10} = \frac{6}{10}$$

$$P[|X-0,5| \le 0,5] = P[\{X<1\} \cup \{X>0\}]$$
$$= P[\{0<X<1\}]$$
$$= 1 - \frac{6}{10} = \frac{4}{10}$$

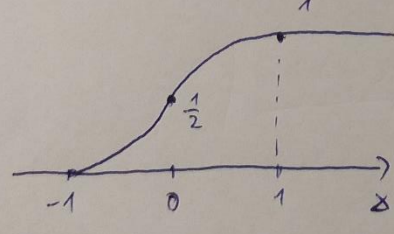
$\boxed{4.17}$

$$1 = c\int_{-1}^{1}(1-x^2)\,dx = c\left[x\Big|_{-1}^{1} - \frac{x^3}{3}\Big|_{-1}^{1}\right] = c\left[2 - \frac{1}{3}\cdot 2\right] = \frac{4}{3}c$$
$$\Rightarrow c = \frac{3}{4}$$

$f_X(x) = \frac{3}{4}(1-x^2)$ với $-1 \le x \le 1$



$$F_X(x) = \frac{3}{4}\int_{-1}^{x}(1-y^2)\,dy$$
$$= \frac{3}{4}\left[y\Big|_{-1}^{x} - \frac{y^3}{3}\Big|_{-1}^{x}\right]$$
$$= \frac{3}{4}\left[(x+1) - \frac{1}{3}(x^3+1)\right]$$



$$P[X=0] = F_X(0) = 0$$

$$P[0<X<0,5] = \frac{3}{4}\left[\left(\frac{1}{2}+1\right) - \frac{1}{3}\left(\frac{1}{8}+1\right)\right] - \frac{3}{4}\left(1-\frac{1}{3}\right)$$
$$= \frac{11}{32}$$

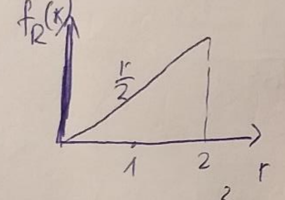
$$P\left[\left|x-\frac{1}{2}\right|<\frac{1}{4}\right] = P\left[\frac{1}{4}<x<\frac{3}{4}\right]$$

$$= \frac{3}{4}\left[\left(\frac{3}{4}+1\right)-\frac{1}{3}\left(\left(\frac{3}{4}\right)^3+1\right)\right] - \frac{3}{4}\left[\left(\frac{1}{4}+1\right)-\frac{1}{3}\left(\left(\frac{1}{4}\right)^3+1\right)\right]$$

$$= 0{,}2734$$

$\boxed{4.19}$

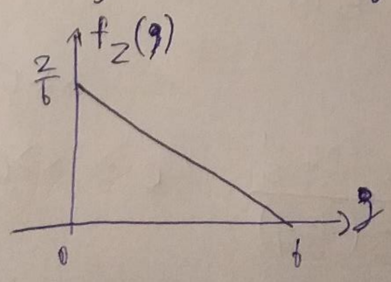
a. $f_R(r) = \frac{d}{dx}F_R(r) = 2\left(\frac{r}{2}\right)\left(\frac{1}{2}\right) = \frac{r}{2}$ với $0\le r\le 2$



b. $P\left[R>\frac{1}{4}\right] = \int_{\frac{1}{4}}^{2} \frac{r}{2}\,dr = \frac{1}{2}\cdot\frac{r^2}{2}\Big|_{\frac{1}{4}}^{2} = \frac{1}{4}\left(4-\frac{1}{16}\right) = \frac{63}{64}$

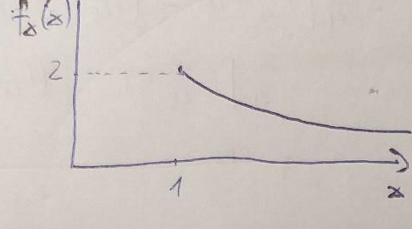
$\boxed{4.20}$

$$f_Z(z) = \frac{d}{dz}F_Z(z) = \frac{2}{b} - \frac{2z}{b^2} = \frac{2}{b}\left(1-\frac{z}{b}\right)$$



$\boxed{4.21}$

$$f_X(x) = \frac{d}{dx} F_X(x) = 2\frac{1}{x^3} \qquad x \geq 1$$



$a > 1$

$$P[X > a] = 2\int_a^{\infty} \frac{1}{x^3}\,dx = \frac{1}{x^2}\Big|_a^{\infty} = \frac{1}{a^2}$$

$$P[X > 2a] = \frac{1}{x^2}\Big|_{2a}^{\infty} = \frac{1}{4a^2}$$

$\boxed{4.30}$

a. $$F_X(x|C) = \frac{P[\{X \leq x\} \cap \{X > 0\}]}{P[X>0]} = \frac{P[0 < X \leq x]}{P[X>0]}$$

$x \geq 0$

$$= \begin{cases} 0 & x \leq 0 \\ \dfrac{F_X(x) - F_X(0)}{1 - F_X(0)} & x \geq 0 \end{cases}$$

$$= \begin{cases} 0 & x \leq 0 \\ \dfrac{-\frac{1}{4}e^{-2x} + \frac{1}{4}}{1 - \left(1 - \frac{1}{4}\right)} = 1 - e^{-2x} & x \geq 0 \end{cases}$$

smartosc

b. $F_X(x|C) = \dfrac{P[\{X \le x\} \cap \{X = 0\}]}{P[X=0]}$

$= \begin{cases} 0 & x<0 \\ 1 & x \ge 0 \end{cases}$

$\boxed{4.32}$

$F_X(x|B) = \dfrac{P[\{X \le x\} \cap \{X > 0,25\}]}{P[X > 0,25]} = \dfrac{P[0,25 < X \le x]}{P[X > 0,25]}$

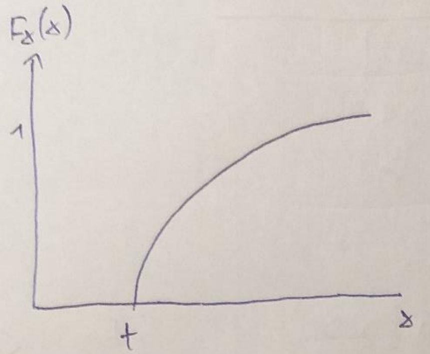
$= \begin{cases} 0 & x < 0,25 \\ \dfrac{F_X(x) - F_X(0,25)}{1 - F_X(0,25)} & x \ge 0,25 \end{cases}$

$= \begin{cases} 0 & x \le 0,25 \\ \dfrac{-\frac{1}{4}e^{-2x} + \frac{1}{4}e^{-2(\frac{1}{4})}}{1-(1-\frac{1}{4}e^{-2(\frac{1}{4})})} = \dfrac{e^{-\frac{1}{2}} - e^{-2x}}{e^{-\frac{1}{2}}} = 1 - e^{-(2x-\frac{1}{2})} & \end{cases}$

$f_X(x|B) = \begin{cases} \dfrac{f_X(x)}{1 - F_X(0,25)} & x \ge 0,25 \\ 0 & x < 0,25 \end{cases}$

$= \begin{cases} \dfrac{\frac{1}{2}e^{-2x}}{\frac{1}{4}e^{-2(\frac{1}{4})}} = 2e^{-(2x-\frac{1}{2})} & x \ge 0,25 \\ 0 & x \le 0,25 \end{cases}$

4.33



$$F_X(x|X>T) = \frac{P[\{X \le x\} \cap \{X > t\}]}{P[X>t]}$$

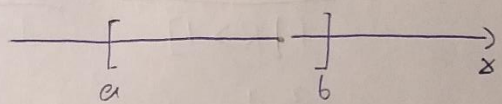
$$= \begin{cases} 0 & x<t \\ \dfrac{F_X(x) - F_X(t)}{1-F_X(t)} & x \geq t \end{cases}$$

$$= \begin{cases} 0 & x<t \\ \dfrac{(1-e^{-\lambda x}) - (1-e^{-\lambda t})}{1-(1-e^{-\lambda t})} & x \geq t \end{cases}$$

$$= \begin{cases} 0 & x<t \\ \dfrac{e^{-\lambda x} - e^{-\lambda t}}{e^{-\lambda t}} & x \geq t \end{cases}$$

$\boxed{4.35}$

$$F_X(x|a \leq X \leq b) = \frac{P[\{X \leq x\} \cap \{a \leq X \leq b\}]}{P[a \leq X \leq b]}$$



$$\Rightarrow \{X \leq x\} \cap \{a \leq X \leq b\} = \begin{cases} \emptyset & x < a \\ \{a \leq X \leq x\} & a \leq x \leq b \\ \{a \leq X \leq b\} & x > b \end{cases}$$

$$\Rightarrow F_X(x|a \leq X \leq b) = \begin{cases} \dfrac{P[\emptyset]}{P[a \leq X \leq b]} = 0 & x < a \\[2ex] \dfrac{P[a \leq X \leq x]}{P[a \leq X \leq b]} = \dfrac{F_X(x) - F_X(a-1)}{F_X(b) - F_X(a^-)} & a \leq x \leq b \\[2ex] \dfrac{P[a \leq X \leq b]}{P[a \leq X \leq b]} = 1 & x > b \end{cases}$$

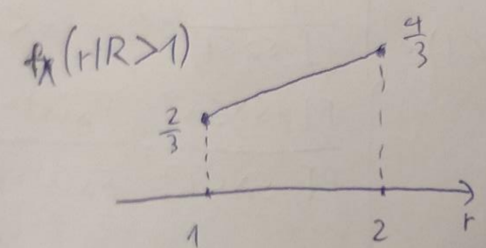
$\boxed{4.36}$

$$F_R(r \mid R>1) = \frac{P[R \le r, R>1]}{P[R>1]} \qquad 1 \le r \le 2$$

$$= \frac{P[1 \le R \le r]}{P[R>1]} = \frac{\left(\frac{r}{2}\right)^2 - \left(\frac{1}{2}\right)^2}{1-\left(\frac{1}{2}\right)^2}$$

$$= \frac{r^2-1}{4-1} = \frac{1}{3}(r^2-1)$$

$$f_R(r \mid R>1) = \frac{d}{dr}\,\frac{1}{3}(r^2-1) = \frac{2}{3}r \qquad 1 \le r \le 2$$



$\boxed{4.38}$

a. $$F_Y(x) = F_Y(x \mid B_0)\,P[B_0] + F_Y(x \mid B_1)\,P[B_1]$$

$$= P[Y \le x \mid X=-1](1-p) + P[Y \le x \mid X=1]\,p$$

$$= P[X+N \le x \mid X=-1](1-p) + P[X+N \le x \mid X=1]\,p$$

$$= P[N \le x+1](1-p) + P[X+N \le x \mid X=1]\,p$$

$$= P[N \le x+1](1-p) + P[N \le x-1]\,p$$

$$= F_N(x+1)(1-p) + F_N(x-1)\,p$$

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$$f_Y(x) = \frac{d}{dx} F_Y(x) = (1-p)\, f_N(x+1) + p f_N(x-1)$$

$$f_Y(x|B_0) = f_N(x+1) = \frac{\alpha}{2} e^{-\alpha|x+1|}$$

$$f_Y(x|B_1) = f_N(x-1) = \frac{\alpha}{2} e^{-\alpha|x-1|}$$

$$f_Y(x) = \frac{1}{2} \left[\frac{\alpha}{2} e^{-\alpha|x+1|} + \frac{\alpha}{2} e^{-\alpha|x-1|} \right]$$

$$= \frac{1}{4} \alpha \left[e^{-\alpha|x+1|} + e^{-\alpha|x-1|} \right]$$

b. $P[Y<0|B_1] = P[x+N<0|x=1] = P[N<-1]$

$$= \frac{\alpha}{2} e^{-\alpha|-1|} = \frac{\alpha}{2} e^{-\alpha}$$

$P[Y \geq 0|B_0] = P[x+N \geq 0|x=-1] = P[N \geq 1]$

$$= \frac{\alpha}{2} e^{-\alpha}$$

c. $P_E = P[Y<0|B_1]\, P[B_1] + P[Y \geq 0|B_0]\, P[B_0]$

$$= 0{,}5 \cdot \frac{\alpha}{2} e^{-\alpha} + 0{,}5 \cdot \frac{\alpha}{2} e^{-\alpha} = \frac{\alpha}{2} e^{-\alpha}$$

$\boxed{4.41}$

$f_R(r) = \frac{r}{2} \qquad 0 \le r \le 2$

$E[R] = \int_0^2 r\left(\frac{r}{2}\right)dr = \frac{1}{2}\int_0^2 r^2 dr = \frac{1}{2}\frac{r^3}{3}\Big|_0^2 = \frac{8}{6} = \frac{4}{3}$

$E[R^2] = \int_0^2 r^2 \frac{r}{2} dr = \frac{1}{2}\left[\frac{r^4}{4}\right]_0^2 = \frac{16}{8} = 2$

$VAR[R] = E[R^2] - E[R]^2 = 2 - \frac{16}{9} = \frac{2}{9}$

$\boxed{4.42}$

$f_Z(z) = \frac{2}{b}\left(1 - \frac{z}{b}\right) \qquad 0 \le z \le b$

$E[Z] = \frac{2}{b}\int_0^b z\left(1 - \frac{z}{b}\right)dz = \frac{2}{b}\left[\frac{z^2}{2} - \frac{1}{b}\frac{z^3}{3}\right]_0^b$

$= \frac{2}{b}\left[\frac{b^2}{2} - \frac{1}{b}\cdot\frac{b^3}{3}\right] = b\left[1 - \frac{2}{3}\right] = \frac{b}{3}$

$E[Z^2] = \frac{2}{b}\int_0^b z^2\left(1 - \frac{z}{b}\right)dz = \frac{2}{b}\left[\frac{z^3}{3} - \frac{1}{b}\frac{z^4}{4}\right]_0^b$

$= 2b^2\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{b^2}{6}$

$VAR[Z] = E[Z^2] - E[Z]^2 = \frac{b^2}{6} - \frac{b^2}{9} = \frac{b^2}{18}$

smartosc

$\boxed{4.43}$

$$f_X(x) = \frac{2}{x^3} \qquad x \geq 1$$

$$E[X] = \int_1^{\infty} x \frac{2}{x^3} dx = \int_1^{\infty} \frac{2}{x^2} dx = -\frac{2}{x}\Big|_1^{\infty} = 2$$

$$E[X^2] = \int_1^{\infty} x^2 \frac{2}{x^3} dx = \int_1^{\infty} \frac{2}{x} dx = 2 \ln x \Big|_1^{\infty} = \infty$$

$\boxed{4.54}$

$$\mathcal{E}[Y] = \int_{-\infty}^{\infty} g(x) f_X(x)\, dx$$

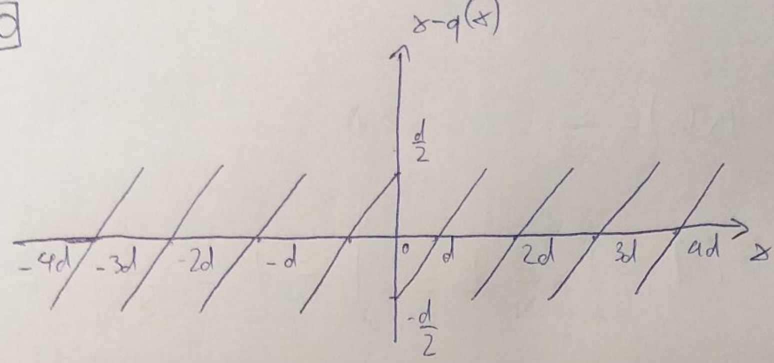
$$= -a \int_{-\infty}^{-a} f_X(x) dx + \int_{-a}^{a} x f_X(x) dx + a \int_{a}^{\infty} f_X(x) dx$$

$$= -a F_X(-a) + \int_{-a}^{a} x f_X(x) dx + a(1 - F_X(a^-))$$

$$\mathcal{E}[Y^2] = a^2 F_X(-a) + \int_{-a}^{a} x^2 f_X(x) dx + a^2(1 - F_X(a^-))$$

$$VAR[Y] = \mathcal{E}[Y^2] - \mathcal{E}[Y]^2$$

$\boxed{4.60}$



Với $-\frac{d}{2} < y < \frac{d}{2}$ thì $y = x - q(x)$ có 8

$$f_y(y) = \sum_{k=1}^{8} \frac{f_x(x_k)}{\frac{dy}{dx}\Big|_{x=x_k}}$$

Vì $x - q(x)$ gồm các đoạn tuyến tính rời rạc

$$\Rightarrow \frac{dy}{dx}\Big|_{x=x_k} = 1 \quad \forall x_k$$

$$\Rightarrow f_y(y) = \sum_{k=1}^{9} f_x(x_k) = \sum_{k=1}^{8} \frac{1}{8d} = \frac{1}{d}$$

với $-\frac{d}{2} < y < \frac{d}{2}$

$\boxed{4.61}$

a. $P[x \le d] = F_x(d) = 1 - e^{-\lambda d} \quad d > 0$

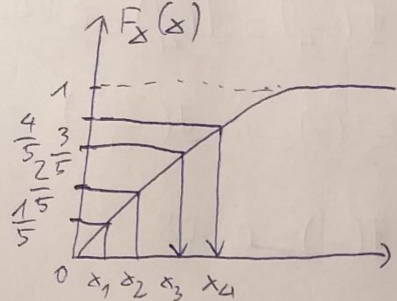
$$P[kd \le x \le (k+1)d] = F_x((k+1)d) - F_x(kd)$$

$$= e^{-\lambda kd} - e^{-\lambda(k+1)d}$$

$$= e^{-\lambda kd}(1 - e^{-\lambda d})$$

$$P[X > kd] = 1 - P[X \le kd] = 1 - F_X(kd) = e^{-\lambda kd}$$

b. $$F_X(x_k) = \frac{k}{5} = 1 - e^{-\lambda x_k}$$



$$x_1 = \frac{\ln\frac{5}{4}}{\lambda}$$

$$x_2 = \frac{\ln\frac{5}{3}}{\lambda}$$

$$x_3 = \frac{\ln\frac{5}{2}}{\lambda}$$

$$x_4 = \frac{\ln 5}{\lambda}$$

$\boxed{4.63}$

a. $P[X>4] = 1 - F_X(4) = 1 - \Phi\left(\frac{4-5}{4}\right)$

$= 1 - \Phi\left(-\frac{1}{4}\right) = \Phi\left(\frac{1}{4}\right) = 0,598$

$P[X \geq 7] = 1 - F_X(7) = 1 - \Phi\left(\frac{7-5}{4}\right)$

$= 1 - \Phi\left(\frac{1}{2}\right) = 0,308$

$P[6,72 < X < 10.16] = \Phi\left(\frac{10.16-5}{4}\right) - \Phi\left(\frac{6.72-5}{4}\right)$

$= \Phi(1,29) - \Phi(0,43) = 0,235$

b. $P[X < a] = 0,8869$

$\Phi\left(\frac{a-5}{4}\right) = 0,8869 = 1 - Q(x)$

$Q(x) = 0,1131 \Rightarrow x = 1,2 = \frac{a-5}{4} \Rightarrow a = 9,8$

c. $P[X > b] = 1 - \Phi\left(\frac{b-5}{4}\right) = 0,11131$

$Q(x) = 0,11131 \rightarrow x = 1,2 = \frac{b-5}{4} \rightarrow b = 9,9$

d. $P[13 < X \leq C] = 0,0123$

$\Phi\left(\frac{c-5}{4}\right) - \Phi\left(\frac{13-5}{4}\right) = \Phi\left(\frac{c-5}{4}\right) - \Phi(2) = 0,0123$

$\Phi\left(\frac{c-5}{4}\right) = 0,0123 + 0,9772 = 0,9895$

$Q\left(\frac{c-5}{4}\right) = 0,0105 \rightarrow x = 2,3 = \frac{c-5}{4} \Rightarrow c = 14,2$

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$\boxed{5.1}$

$$P[X=0, Y=0] = P[\{00\}] = \frac{1}{16}$$

$$P[X=0, Y=1] = P[\{01, 10\}] = \frac{1}{4}$$

$$P[X=0, Y=2] = P[\{02, 20\}] = \frac{1}{8}$$

$$P[X=1, Y=1] = P[\{11\}] = \frac{1}{4}$$

$$P[X=1, Y=2] = P[\{12, 21\}] = \frac{1}{4}$$

$$P[X=2, Y=2] = P[\{22\}] = \frac{1}{16}$$

$$P[X=Y] = P[(X,Y) \in \{00, 11, 22\}] = \frac{1}{16} + \frac{1}{4} + \frac{1}{16} = \frac{3}{8}$$

$\boxed{5.3}$

a. $S_{XY} = \{(-1,-1), (-1,0), (-1,1), (1,-1), (1,0), (1,1)\}$

b. $P[X=1, Y=-1] = P[Y=-1 | X=1]\, P[X=1] = \frac{3}{4} p$

$P[X=-1, Y=-1] = \frac{1}{4}(1-p-p_e)$

$P[X=1, Y=0] = P[Y=0 | X=1]\, P[X=1] = \frac{3}{4} p_e$

$P[X=-1, Y=0] = \frac{1}{4} p_e$

$P[X=1, Y=1] = P[Y=1 | X=1][X=1] = \frac{3}{4}(1-p-p_e)$

$P[X=-1, Y=1] = \frac{1}{4} p$

c. $P[X \neq Y] = \frac{1}{4} p_e + \frac{1}{4} p + \frac{3}{4} p_e + \frac{3}{4} p$

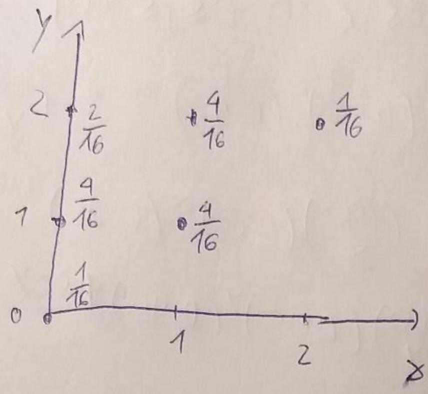
$= p_e + p$

$P[Y=0] = \frac{3}{4} p_e + \frac{1}{4} p_e$

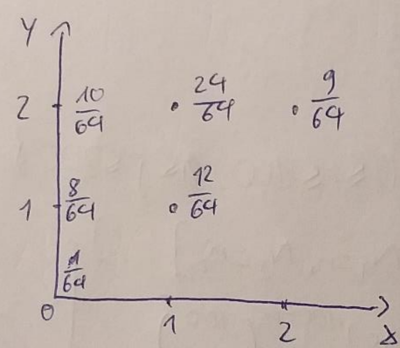
$= p_e$

$\boxed{5.9}$

a.


y
2
2/16
4/16
1/16
1
4/16
4/16
1/16
0
1
2
x

b.


y
2
10/64
24/64
9/64
1
8/64
12/64
1/64
0
1
2
x

$\boxed{5.13}$

a. $S_{XY} = \{(i,j): 0 \le i \le 100;\ 0 \le j \le 100\}$

b. $P_{XY}(x,y) = \binom{100}{x}(0,05)^{x}(0,95)^{100-x} \cdot \binom{100}{y}(0,05)^{y}(0,95)^{100-y}$

c. $P[X=x] = \binom{100}{x}(0,05)^{x}(0,95)^{100-x}$

$P[Y=y] = \binom{100}{y}(0,05)^{y}(0,95)^{100-y}$

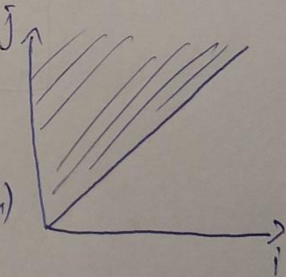
$\boxed{5.14}$

a. $S_{XY} = \{(i,j): 0 \le i \le 100;\ i \le j \le 200\}$

b. $P_{N_1N_2}(n_1,n_2) = P[N_1=n_1, N_2=n_2]$

$= \binom{100}{n_1}(0,05)^{n_1}(0,95)^{100-n}\binom{100}{n_2-n_1}$

$(0,05)^{n_2-n_1}(0,95)^{100-(n_2-n_1)}$



c. $P_{N_1}(n_1) = \binom{100}{n_1}(0,05)^{n_1}(0,95)^{100-n_1}$

$P_{N_2}(n_2) = \binom{200}{n_2}(0,05)^{n_2}(0,95)^{200-n_2}$

d. $P[A] = P[N_1 < N_2] = 1 - P[N_1 = N_2]$ $(N_1 \le N_2)$

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$\boxed{5.17}$

$$P[X\le x] = F_{XY}(x,\infty) = x^2$$
$$P[Y\le y] = F_{XY}(\infty,y) = 2\left(y-\frac{y^2}{2}\right)$$

$$P\left[X\le\frac{1}{2}, Y\le\frac{3}{4}\right] = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$P\left[\frac{1}{4}<X\le\frac{3}{4}, \frac{1}{4}<Y\le\frac{3}{4}\right]$$
$$= F_{XY}\left(\frac{3}{4},\frac{3}{4}\right) - F_{XY}\left(\frac{3}{4},\frac{1}{4}\right) - F_{XY}\left(\frac{1}{4},\frac{3}{4}\right) + F_{XY}\left(\frac{1}{4},\frac{1}{4}\right)$$

$$= \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{4}\cdot\frac{1}{4} - \frac{1}{2}\left(\frac{1}{4}\right)^2\right) - \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{1}{4}$$

$\boxed{5.25}$

a. $x>0, y>0$

$$F_{XY}(x,y) = \int_0^x\int_0^y \frac{1}{2}e^{-\frac{x}{2}}\, 2ye^{-y^2}\,dx\,dy$$
$$= \left(1-e^{-\frac{x}{2}}\right)\left(1-e^{-y^2}\right)$$

b. $$P[X>Y] = \int_0^\infty\int_0^x 2ye^{-y^2}\,dy\,\frac{1}{2}e^{-\frac{x}{2}}\,dx$$
$$= \int_0^\infty \left(1-e^{-x^2}\right)\frac{1}{2}e^{-\frac{x}{2}}\,dx$$

$$= 1 - \frac{1}{2}\int_0^{\varphi} e^{-\left(x^2+\frac{x}{2}\right)} dx$$

$$= 1 - \frac{1}{2} e^{\frac{1}{16}} \int_0^{\varphi} e^{-\left(x+\frac{1}{4}\right)^2} dx$$

$$= 1 - \frac{\sqrt{\pi}\, e^{\frac{1}{16}}}{2}$$

$\boxed{5.28}$

a. $k\pi = 1 \Rightarrow k = \frac{1}{\pi}$ ①

$k\sqrt{2}\sqrt{2} = 1 \Rightarrow k = \frac{1}{2}$ ②

$k.\frac{1^2}{2} = 1 \Rightarrow k = 2$ ③

b. ① $f_x(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} = \frac{2\sqrt{1-x^2}}{\pi}$ $\quad -1 < x < 1$

$f_y(y) = \frac{2\sqrt{1-y^2}}{\pi}$ $\quad -1 < y < 1$

② $f_x(x) = \int_{|x|-1}^{1-|x|} \frac{dy}{2} = 1 - |x|$ $\quad -1 < x < 1$

$f_y(y) = 1 - |y|$ $\quad -1 < y < 1$

③ $f_x(x) = \int_0^{1-x} 2dy = 2(1-x)$ $\quad 0 < x < 1$

$f_y(y) = 2(1-y)$ $\quad 0 < y < 1$

$\boxed{5.39}$

a. $P_{XY}(r,r) = 0$

$P_X(r) = \frac{1}{8}$

$P_Y(r) = \frac{1}{8}$

$\Rightarrow P_{XY}(r,r) \neq P_X(r) . P_Y(r)$

X và Y không độc lập

b.

$X \backslash Y$	$-r$	$\frac{-r}{\sqrt{2}}$	0	$\frac{r}{\sqrt{2}}$	r
$-r$	0	0	$\frac{1}{6}$	0	0
$\frac{-r}{\sqrt{2}}$	0	$\frac{1}{12}$	0	$\frac{1}{12}$	0
0	$\frac{1}{6}$	0	0	0	$\frac{1}{6}$
$\frac{r}{\sqrt{2}}$	0	$\frac{1}{12}$	0	$\frac{1}{12}$	0
r	0	0	$\frac{1}{6}$	0	0

$P_X(-r) = \frac{1}{6}; P_X\left(\frac{-r}{\sqrt{2}}\right) = \frac{1}{6}; P_X(0) = \frac{1}{3}; P_X\left(\frac{r}{\sqrt{2}}\right) = \frac{1}{6};$

$P_X(r) = \frac{1}{6}$

$P_Y(-r) = \frac{1}{6}; \; P_Y\left(\frac{-r}{\sqrt{2}}\right) = \frac{1}{6}; \; P_Y(0) = \frac{1}{3}; \; P_Y\left(\frac{r}{\sqrt{2}}\right) = \frac{1}{6}; P_Y(r) = \frac{1}{6}$

$P_{XY}(0,0) = 0$

$P_X(0) . P_Y(0) = \frac{1}{9}$

$\Rightarrow$ X và Y không độc lập

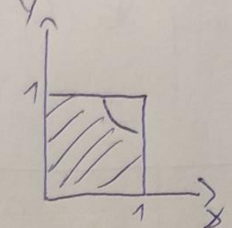
$\boxed{5.98}$

a. $P[x^2 < \frac{1}{2}, |Y| < \frac{1}{2}] = P[x^2 < \frac{1}{2}] P[|Y| < \frac{1}{2}]$

$= P[x < \frac{1}{\sqrt{2}}] P[Y < \frac{1}{2}] = \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

b. $P[4x < 1, Y < 0] = P[x < \frac{1}{4}] P[Y < 0] = \frac{1}{4} \cdot 0 = 0$

c.



$f(x,y) = f(x) f(y) = 1$

$P[xY < \frac{1}{2}] = \frac{1}{2} + \int_{\frac{1}{2}}^{1} \int_{0}^{\frac{1}{2x}} 1 \, dy \, dx$

$= \frac{1}{2} + \int_{\frac{1}{2}}^{1} \frac{1}{2x} \, dx$

$= \frac{1}{2} + \frac{1}{2} \ln|x| \Big|_{\frac{1}{2}}^{1}$

$= 0{,}85$

d. $P[\min(x,Y) > \frac{1}{3}] = P[x > \frac{1}{3}] P[Y > \frac{1}{3}]$

$= (\frac{2}{3})^2 \cdot \frac{4}{9}$

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$\boxed{5.56}$

a. $\mathcal{E}[(X+Y)^2] = \mathcal{E}[X^2 + 2XY + Y^2]$

$= \mathcal{E}[X^2] + 2\mathcal{E}[XY] + \mathcal{E}[Y^2]$

b. $VAR[X+Y] = \mathcal{E}[(X+Y)^2] - \mathcal{E}[X+Y]^2$

$= \mathcal{E}[X^2] + 2\mathcal{E}[XY] + \mathcal{E}[Y^2] - \mathcal{E}[X]^2$

$- 2\mathcal{E}[X]\mathcal{E}[Y] - \mathcal{E}[Y]^2$

$= VAR[X] + VAR[Y] + 2[\mathcal{E}[XY] -$

$\mathcal{E}[X]\mathcal{E}[Y]]$

$\boxed{5.72}$

$$E[XY] = \int_0^{2\pi} \cos\frac{\theta}{4} \sin\frac{\theta}{4} \frac{d\theta}{2\pi}$$

$$= \frac{1}{4\pi} \int_0^{2\pi} \sin\frac{\theta}{2} d\theta = \frac{1}{4\pi} \left[-2\cos\frac{\theta}{2}\right]_0^{2\pi}$$

$$= \frac{2}{4\pi}\left[\cos 0 - \cos\frac{2\pi}{2}\right]$$

$$= \frac{4}{4\pi} = \frac{1}{\pi}$$

$\boxed{5.87}$

$$P[K=k|N=n] = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P[K=k] = \sum_{n=\max(k,1)}^{\infty} P[K=k|N=n]\, P[N=n]$$

chỉ nhận được $k>0$ tin khi $N \geq k$

$$P[K=0] = \sum_{n=1}^{\infty} (1-p)^n (1-a) a^{n-1}$$

$$= (1-p)(1-a) \sum_{n=1}^{\infty} [(1-p)a]^{n-1}$$

$$= \frac{(1-p)(1-a)}{1-(1-p)a}$$

Với $k \geq 1$:

$$P[K=k] = \sum_{n=k}^{\infty} \binom{n}{k} p^k (1-p)^{n-k} (1-a) a^{n-1}$$

$$= \frac{(1-a) p^k a^k}{a} \sum_{n=k}^{\infty} \binom{n}{k} [(1-p)a]^{n-k}$$

$$= \frac{(1-a) p^k a^k}{a(1-(1-p)a)^{k+1}}$$

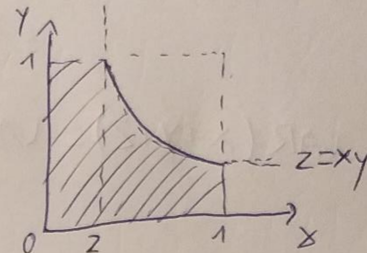
$$= \frac{(1-a)}{a(1-(1-p)a)} \left(\frac{pa}{1-(1-p)a} \right)^k$$

smartosc

$\boxed{5.95}$

$Z = XY \quad f_{XY}(x,y) = 1 \qquad 0 \le x \le 1, \quad 0 \le y \le 1$

$$F_Z(z) = P[Z \le z] = P[XY \le z] = P\left[Y \le \frac{z}{x}\right]$$



$$= z + \int_z^1 \int_0^{\frac{z}{x}} dy\,dx$$

$$= z + \int_z^1 \frac{z}{x}\,dx$$

$$= z + \left[z \ln x\right]_z^1$$

$$= z - z\ln z$$

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

$$= 1 - \ln z - \frac{z}{z}$$

$$= \begin{cases} -\ln z & 0 \le z \le 1 \\ 0 & \text{TH còn lại} \end{cases}$$

$\boxed{7.1}$

a. $VAR(X+Y+Z) = VAR(X) + VAR(Y) + VAR(Z) + 2COV(X,Y) + 2COV(X,Z) + 2COV(Y,Z)$

$= 1+1+1+2\left(\frac{1}{4}\right)+2.0+2\left(\frac{-1}{4}\right) = 3$

b. $VAR(X+Y+Z) = VAR(X) + VAR(Y) + VAR(Z) = 3$

$\boxed{7.2}$

$$\mathcal{E}[S_n] = \mathcal{E}\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} \mathcal{E}[X_i] = n\mu$$

$$VAR(S_n) = \sum_{k=1}^{n} VAR(X_k) + \sum_{j=1}^{n}\sum_{k=1}^{n} cov(X_j, X_k)$$

$$K = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & 0 & & \cdots & 0 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & 0 & \cdots & 0 \\ & & \cdots & & & 0 \\ & & \cdots & & & \rho\sigma^2 \\ & & \cdots & & & \sigma^2 \end{bmatrix}$$

$$VAR(S_n) = n\sigma^2 + 2(n-1)\rho\sigma^2$$

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$\boxed{7.3}$

$$\mathcal{E}[S_n] = n\mu$$

$$K = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & \rho^2\sigma^2 & \dots & \rho^{n-1}\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & \dots & \rho^{n-2}\sigma^2 \\ \vdots & & & & \\ \rho^{n-1}\sigma^2 & & \dots & & \sigma^2 \end{bmatrix}$$

$$VAR(S_n) = n\sigma^2 + 2\rho\sigma^2 \sum_{j=1}^{n-1} \sum_{k=0}^{j-1} \rho^k$$

$$= n\sigma^2 + 2\rho\sigma^2 \sum_{j=1}^{n-1} \frac{1-\rho^j}{1-\rho}$$

$$= n\sigma^2 + 2\rho\sigma^2 \left[\frac{n-1}{1-\rho} - \left(\frac{\rho}{1-\rho}\right)\frac{1-\rho^{n-1}}{1-\rho}\right]$$

$\boxed{7.4}$

a. $\Phi_Z(\omega) = \Phi_X(\omega)\,\Phi_Y(\omega) = e^{-\alpha|\omega|} e^{-\beta|\omega|} = e^{-(\alpha+\beta)|\omega|}$

b. $f_Z(z) = \Phi_Z^{-1}(\omega) = \frac{1}{\lambda} \cdot \frac{\alpha+\beta}{(\alpha+\beta)^2 + z^2} \Rightarrow Z$