Date ....

Chương 2 :

2.1, a, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

b, $A = \{1, 2, 3, 4\}$ $B = \{2, 3, 4, 5, 6, 7, 8\}$

$D = \{1, 3, 5, 7, 9, 11\}$

c, $A \cap B \cap D = \{3\}$ $A^c \cap B = \{5, 6, 7, 8\}$

$A \cup (B \cap D^c) = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B) \cap D^c = \{2, 4, 6, 8\}$

2.2, a, (x, y)

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

b, $A = \{(N_1 < N_2)^c\} = \{N_1 \geq N_2\} =$
$\{(1,1); (2,1); (2,2); (3,1); (3,2); (3,3); (4,1); (4,2);$
$(4,3); (4,4); (5,1); (5,2); (5,3); (5,4); (5,5); (6,1);$
$(6,2); (6,3); (6,4); (6,5); (6,6)\}$

e, $A \cap B^c = \{N_2 \leq N_1 < 6\} =$
$\{(1,1); (2,1); (2,2); (3,1); (3,2); (3,3); (4,1); (4,2); (4,3)$
$(4,4); (5,1); (5,2); (5,3); (5,4); (5,5)$

f, $C = \{(3,1); (4,2); (5,3); (6,4); (1,3);$
$(2,4); (3,5); (4,6)\}$

$A \cap C = \{(3,1); (4,2); (5,3); (6,4)\}$

2.4, a, $(2,0); (2,1); (2,2); (-2,-2); (-2,-1)$
$(-2,0)$

b, $\{(2,1); (2,2)\}$

c. $\{y = 0\} = \{(2,0); (-2,0)\}$

No. ....
Date ....

2.15, a, $D = A_1 \cap A_2 \cap A_3$

b, $D = A_1 \cup A_2 \cup A_3$

c, $D = (A_1 \cap A_2 \cap A_3) \cup (A_1^c \cap A_2 \cap A_3) \cup (A_1 \cap A_2^c \cap A_3)$
$\cup (A_1 \cap A_2 \cap A_3^c)$

2.23, $P[\{c, d\}] = P_c + P_d = 3/8$

$P[\{b, c\}] = P_b + P_c = 6/8$

$P[\{d\}] = 1/8 = P_d$

$P_a + P_b + P_c + P_d = 1$

$\Rightarrow P_a = 1/8$; $P_b = 4/8$; $P_c = 2/8$; $P_d = 1/8$

2.34, $P[I] = k \text{ length}(I)$

$I = [-1, 2]$

$1 = P[S] = P[(-1, 2)] = k \text{ length}((-1, 2)) = 3k$

$\Rightarrow k = 1/3$

a, $P[A] = \frac{1}{3} \text{ length}((-1, 0)) = \frac{1}{3}(1) = \frac{1}{3}$

$P[B] = \frac{1}{3} \text{ length}((-0{,}5, 1)) = \frac{1}{3} \cdot \frac{3}{2} = \frac{1}{2}$

$P[C] = \frac{1}{3} \text{ length}((0{,}75, 2)) = \frac{1}{3} \cdot \frac{5}{4} = \frac{5}{12}$

$P[A \cap B] = \frac{1}{3} \text{ length}((-0{,}5, 0)) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$P[A \cap C] = P[\emptyset] = 0$

b, $P[A \cup B] = \frac{1}{3} \text{ length}((-1, 1)) = \frac{2}{3}$

$P[A \cup C] = \frac{1}{3} \text{ length}(A \cup C) = \frac{1}{3}\left(1 + \frac{5}{4}\right) = \frac{3}{4}$

$P[A \cup B \cup C] = P(S) = 1$

ẢI TIẾN $P[A \cup B] = P[A] + P[B] - P[A \cap B] = \frac{2}{3}$

$P[A \cup C] = P[A] + P[C] - P[A \cap C] = \frac{3}{4}$

$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C]$
$+ P[A \cap B \cap C]$
$= \frac{1}{3} + \frac{1}{2} + \frac{5}{12} - \frac{1}{6} - 0 - \frac{1}{12} + 0 = 1$

2.36, $A = \{x > 4\}$ $B = \{x > 8\}$

a, $P[A \cap B] = P[\{x > 4\} \cap \{x > 8\}] = P[\{x > 8\}] = 1/12$

$P[A \cup B] = P[\{x > 4\} \cup \{x > 8\}] = P[\{x > 4\}] = 1/4$

b, $P[\{x > 6\}] = P[\{6 < x \leq 12\} \cup \{x > 12\}]$
$= P[\{6 < x \leq 12\}] + P[\{x > 12\}]$

$\Rightarrow P[\{6 < x \leq 12\}] = 1/6 - 1/12 = 1/12$

$\boxed{\text{Chương 3}}$

3.5, a, eperiment i

	A_i	A_i^c
B_i	1/4	1/4
B_i^c	1/4	1/4

$S = S_1, S_2, \ldots$ S_i là 1 outcome từ eperiment cơ bản

b, $X(S) = n$

với n là lần đầu tiên xuất hiện của $A_i B_i^c \cap A_i^c B_i$ trong $S_1, S_2,$

c, $P[A_i B_i^c \cup A_i^c B_i] = P[A_i B_i^c] + P[A_i^c B_i] = \frac{1}{2}$
$= P[\text{ssuccess}]$

$P[X = k] = P[(k-1) \text{ failues}, 1 \text{ successess}] = (1/2)^k$

3.9, a, $Y = n - m - m = n - 2m$ $0 \leq m \leq n$

$S_Y = \{-n; -n+2, \ldots n-2; n\}$

b, $P[Y = 0] = P[n = 2m] = P[m = \frac{n}{2}]$

$P[Y = k] = P[n - 2m = k] = P[m = \frac{n-k}{2}]$

TIẾN

3.12, a, $1 = P_1 + P_2 + P_3 + P_4 = P_1\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \Rightarrow P_1 = \frac{12}{25}$

$P_2 = \frac{6}{25}$; $P_3 = \frac{4}{25}$; $P_4 = \frac{3}{25}$

b, $1 = P_1\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)$.

$\Rightarrow P_1 = \frac{8}{15}$; $P_2 = \frac{4}{15}$; $P_3 = \frac{2}{15}$; $P_4 = \frac{1}{15}$

c, $1 = P_1\left(1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{64}\right) = \frac{105}{64} P_1$

$\Rightarrow P_1 = \frac{64}{105}$; $P_2 = \frac{32}{105}$; $P_3 = \frac{8}{105}$; $P_4 = \frac{1}{105}$

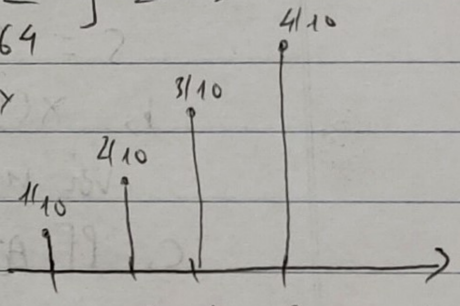
3.13, a, $1 = \sum_{k=1}^{\infty} \frac{c}{k^2} = c \sum_{k=1}^{\infty} \frac{1}{k^2}$

$\Rightarrow c = 0,608$

b, $P[X > 4] = 1 - P[X \le 3] = 1 - c\left[1 + \frac{1}{4} + \frac{1}{9}\right]$
$= 0,1675$

c, $P[6 \le X \le 8] = c\left[\frac{1}{36} + \frac{1}{44} + \frac{1}{64}\right] = 0,39$

3.17, a, $P[Y = 0 + 2] = 4/10$
$P[Y = -1 + 2] = 3/10$
$P[Y = -2 + 2] = 2/10$
$P[Y = -3 + 2] = 1/10$



b, $P[Y = 2] = 4/10$

c, $P[Y > 0] = P[Y = 2] + P[Y = 1] = \frac{7}{10}$

3.22, a, $E[X] = 1 \cdot \frac{12}{25} + 2 \cdot \frac{6}{25} + 3 \cdot \frac{4}{25} + 4 \cdot \frac{3}{25} = \frac{48}{25} = 1,92$

$E[X^2] = 1 \cdot \frac{12}{25} + 4 \cdot \frac{6}{25} + 9 \cdot \frac{4}{25} + 16 \cdot \frac{3}{25} = \frac{120}{25}$

$$VAR[X] = \frac{126}{25} - \left(\frac{48}{25}\right)^2 = \frac{696}{625}$$

b, $$E[X] = 1 \cdot \frac{8}{15} + 2 \cdot \frac{4}{15} + 3 \cdot \frac{2}{15} + 4 \cdot \frac{1}{15} = \frac{26}{15}$$

$$E[X^2] = 1 \cdot \frac{8}{15} + 4 \cdot \frac{4}{15} + 9 \cdot \frac{2}{15} + 16 \cdot \frac{1}{15} = \frac{58}{15}$$

$$VAR[X] = \frac{58}{15} - \left(\frac{26}{15}\right)^2 = \frac{194}{225}$$

c, $$E[X] = 1 \cdot \frac{64}{105} + 2 \cdot \frac{32}{105} + 3 \cdot \frac{8}{105} + 4 \cdot \frac{1}{105} = \frac{156}{105}$$

$$E[X^2] = 1 \cdot \frac{64}{105} + 4 \cdot \frac{32}{105} + 9 \cdot \frac{8}{105} + 16 \cdot \frac{1}{105} = \frac{280}{105}$$

$$VAR[X] = \frac{280}{105} - \left(\frac{156}{105}\right)^2 = \frac{5064}{(105)^2} = 0,459$$

3,31, a, $P[X = k] = \binom{n}{k}\left(\frac{1}{2}\right)^n$

$$E[aX^2 + bX] = a\,E[X^2] + b\,E[X]$$

$$E[X] = \sum_{j=0}^{n} j \binom{n}{j}\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n \sum_{j=0}^{n} j \frac{n!}{j!\,(n-j)!}$$

$$= \left(\frac{1}{2}\right)^n \sum_{j=1}^{n} \frac{n!}{(j-1)!\,(n-j)!}$$

đặt $j' = j - 1$

$$= \left(\frac{1}{2}\right)^n n \sum_{j'=0}^{n-1} \frac{(n-1)!}{j'!\,(n-1-j')!} = n\left(\frac{1}{2}\right)^n \sum_{j'=0}^{n-1} \binom{n-1}{j'}$$

$$= n\left(\frac{1}{2}\right)^n 2^{n-1} = \frac{n}{2}$$

$$E[X^2] = \sum_{j=0}^{n} j^2 \binom{n}{j}\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^n n \sum_{j=1}^{n} j \frac{(n-1)!}{(j-1)!\,(n-j)!}$$

$$= n\left(\frac{1}{2}\right)\left[\sum_{j'=0}^{n-1} j' \binom{n-1}{j'}\left(\frac{1}{2}\right)^{n-1} + \sum_{j'=0}^{n-1} \binom{n-1}{j'}\left(\frac{1}{2}\right)^{n-1}\right.$$

$$= \frac{n}{2}\left(\frac{n}{2} + 1\right)$$

$$E[ax^2 + bx] = a\frac{n}{2}\left(\frac{n}{2}+1\right) + b\frac{n}{2}$$

b, $$E[a^x] = \sum_{j=0}^{n} a^j \binom{n}{j}\left(\frac{1}{2}\right)^j = \sum_{j=0}^{n}\binom{n}{j}\left(\frac{a}{2}\right)^j$$

$$= \left(1+\frac{a}{2}\right)^n$$

3.33, a) $$E[(x-10)^+] = \sum_{i=1}^{15}(i-10)\,P[x=i]$$

$$= p_i \sum_{i=1}^{15}(i-10)\frac{1}{i} = 0,334$$

b, $$E[(x-10)^+] = p_i\sum_{i=1}^{15}(i-10)\,2^{-(i-1)} = 0,00174$$

c, $$E[(x-10)^+] = p_i\sum_{i=1}^{15}(i-10)\,2^{-(i-1)/2} = 1,69\cdot 10^{-17}$$

3.50, $$\frac{p_k}{p_{k-1}} = \frac{\binom{n}{k}p^k q^{n-k}}{\binom{n}{k-1}p^{k-1}q^{n-k+1}} = \frac{\frac{n!}{k!(n-k)!}}{\frac{n!}{(k-1)!}}\cdot\frac{p}{q}$$

$$= \frac{(n-k+1)p}{kq} = \frac{(n+1)p - k(1-q)}{kq} = 1 + \frac{(n+1)p - k}{kq}$$

3.52, $p = 0,01$

a, $$P[N=k] = (1-p)^k p, \quad k = 0,1,2$$

b, $$E[N] = \sum_{k=0}^{\infty} k(1-p)^k p = (1-p)p\sum_{k=0}^{\infty}k(1-p)^{k-1}$$

$$= (1-p)p\frac{1}{(1-(1-p))^2} = \frac{1-p}{p}$$

No. ....
Date ....

c, $0{,}99 = \cancel{P[X\ldots]}\ P[N > k_0] = \sum_{k_0+1}^{\infty} (1-p)^k p^{\cancel{a}}$

$= p(1-p)^{k_0+1} \sum_{k=0}^{\infty} (1-p)^k = (1-p)^{1001}$ $\cancel{\Rightarrow p = 1{,}004}$

$\Rightarrow p = 1{,}004 \cdot 10^{-5}$

3.55. $P[m \geq k] = P[m \geq k+j \mid m > j]$

$= \dfrac{P[M \geq k+j]}{P[M > j]} = \dfrac{P[M \geq k+j]}{P[M \geq j+1]}$

$\Rightarrow P[M \geq k+j] = P[M \geq k]\, P[M \geq j+1]$

$a_k = P[M \geq k]$

$a_{k+j} = a_k a_{j+1}$ (1) $j \geq 1, k \geq 1$

$a_1 = 1, \ a_2 = 1 - P[m = 1] = 1 - p$

~~Rà~~ thay $j = 1$ vào (1) : $a_{k+1} = a_2 a_k \quad k \geq 1$

$a_k = a_2^{k-1} \quad k \geq 1$

$\Rightarrow P[M \geq k] = (1-p)^{k-1}, \quad k \geq 1$

$P[m = k] = P[M \geq k] - P[M \geq k+1]$

$= (1-p)^{k-1} - (1-p)^k$

$= (1-p)^{k-1}(1 - (1-p))$

$= (1-p)^{k-1} p$

3.59, $\lambda = 6000$ r/m $= 100$ r/s

$\alpha = \lambda \cdot \dfrac{1}{10} = 10$ r/100 ms

a, $P[N = 0] = e^{-10} = 4{,}54 \cdot 10^{-5}$

b, $P[5 \leq N \leq 10] = \sum_{k=5}^{10} \dfrac{10^k}{k!} e^{-10} = 0{,}554$

TIẾN

3.67, $p = 10^{-6}$, $n = 10^4$, $np = 10^{-2}$

a, $P[N=0] = e^{-np} = 0,99$

$$P[N \le 3] = \sum_{k=0}^{3} \frac{(0,01)^k}{k!} e^{-np} \approx 1$$

b, $0,99 \quad P[N \ge 1] = 1 - P[N=0] = 1 - e^{-np}$

$$0,01 = e^{-np}$$

$$\Rightarrow p = \frac{\ln 100}{n} = 4,6 \cdot 10^{-6}$$

3.70, $p_k = \frac{1}{c_{10}} \frac{1}{k}$, $k = 1, \dots 10$ $\quad c_{10} = 2,93$

$$p_1 = \frac{1}{2,93}$$

$$P[X > 5] = \frac{1}{c_{10}} \left[\frac{1}{6} + \dots + \frac{1}{10} \right] = 0,2204$$

3.72, $p_k = \frac{1}{c_L} \cdot \frac{1}{k}$

$$\ln p_k = \ln \frac{1}{c_L} + \ln \frac{1}{k}$$

$$= -\ln k - \ln c_L$$

3.74, $p_k = \frac{1}{c_L} \frac{1}{k}$ $\quad L = 10^4$, $c_L = \ln 10^4 + 0,5672$

$$= 9,7876$$

$$0,99 = P[X \le k_0] = \frac{1}{c_{10000}} \sum_{j=1}^{k_0} \frac{1}{j} = \frac{c_{k_0}}{c_{10000}}$$

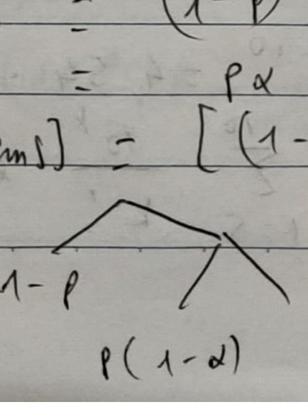
$$= \frac{\ln k_0 + 0,57721}{9,7876} \Rightarrow \ln k_0 = 9,067$$

3.77, a, $P[\text{pass}] = (1-p) + p(1-\alpha)$

$$P[\text{fail}] = p\alpha$$

$$P[k \text{ items}] = \left[(1-p) + p(1-\alpha)\right]^{k-1} (p\alpha)^1$$

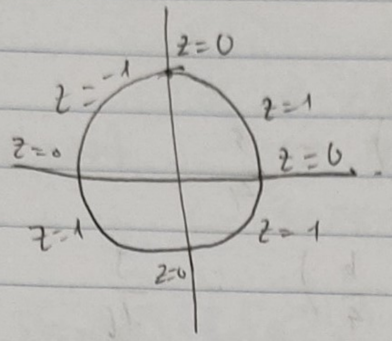
b,



Date ....

Chương 4

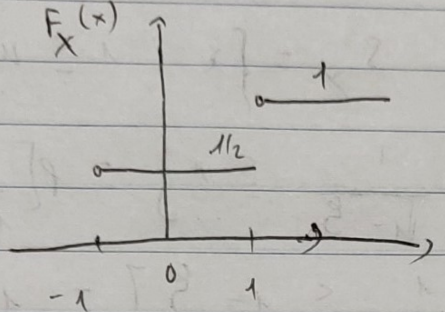
4.3,

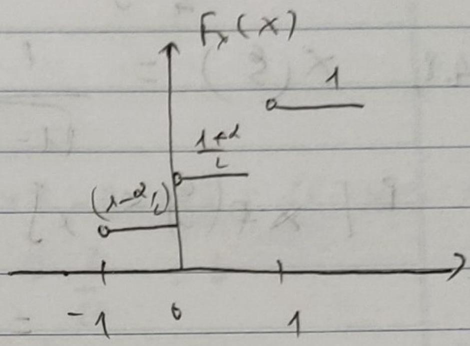


$S_X = \{-1; 0; 1\}$

$P = \left(\frac{1}{4}+\frac{1}{4}, 0, \frac{1}{4}+\frac{1}{4}\right)$

$= \left(\frac{1}{2}, 0, \frac{1}{2}\right)$



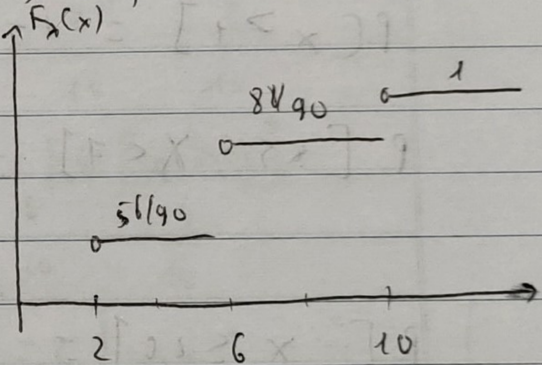


4.4, $S_X = \{2, 6, 10\}$ $S_Y = \{2, 6, 10\}$

X: $P[2] = \frac{8}{10} \cdot \frac{7}{9} = \frac{56}{90}$

$P[6] = 2 \cdot \frac{8}{10} \cdot \frac{2}{9} = \frac{32}{90}$

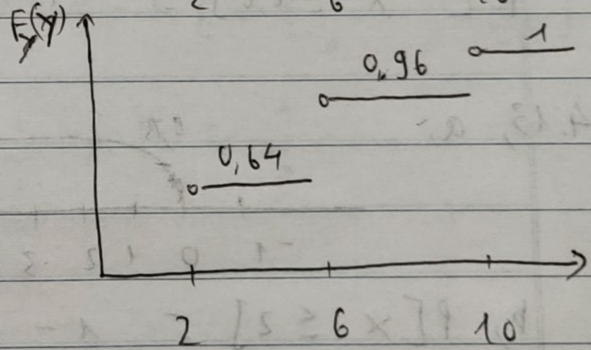
$P[10] = \frac{2}{10} \cdot \frac{1}{9} = \frac{2}{90}$



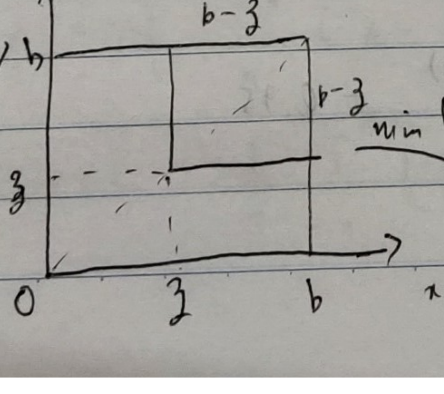
Y: $P[2] = 0{,}8^2 = 0{,}64$

$P[6] = 2 \cdot 0{,}8 \cdot 0{,}2 = 0{,}32$

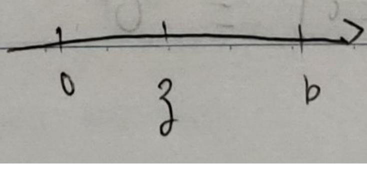
$P[10] = 0{,}2^2 = 0{,}04$



4.7, b



$\min(x,y) = 3$



$S = \{(x,y) : 0 \le x \le b,\ 0 \le y \le b\}$

$S_Z = \{3 : 0 \le 3 \le b\}$

$P(Z \le 3) = 1 - \left(\frac{b-3}{b}\right)^2 = \frac{2\,3}{b} - \frac{3^2}{b^2}$

$P[Z > 0] = 1$

$P[Z > b] = 1$

$P[Z \le \frac{b}{2}] = F_Z\left(\frac{b}{2}\right) = \frac{3}{4}$

$P[Z > \frac{b}{4}] = 1 - F_Z\left(\frac{b}{4}\right) = \frac{9}{16}$

4.8) $X(\xi) = \frac{1}{\sqrt{1-\xi^2}}$ $\quad S_X = \{x : 1 \le x \le \infty\}$

$P[X(\xi) \le x] = P\left[\frac{1}{\sqrt{1-\xi}} \le x\right] = P\left[\frac{1}{1-\xi} \le x^2\right]$

$= P\left[\frac{1}{x^2} \le 1-\xi\right] = 1 - \frac{1}{x^2}$

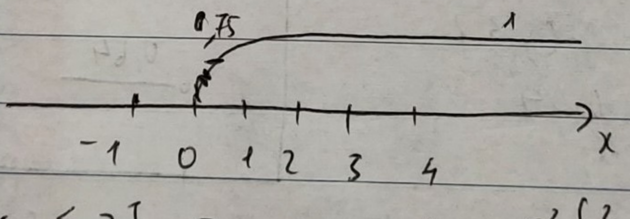
$P[X \ge 1] = 1 - F_X(1) = 1$

$P[5 < X < 7] = F_X(7) - F_X(5) = \left(1 - \frac{1}{49}\right) - \left(1 - \frac{1}{25}\right)$

$= 0,01959$

$P[X \le 20] = 1 - \frac{1}{400} = 0,9975$

4.13, a,



b, $P[X \le 2] = 1 - \frac{1}{4} e^{-2(2)} = 0,9954$

$P[X = 0] = 1 - \frac{1}{4} e^{-2(0)} = 0,75$

$P[X < 0] = 0$

$P[2 < X < 6] = P[X \le 6] - P[X \le 2]$

$= 1 - \frac{1}{4} e^{-2(6)} - 1 + \frac{1}{4} e^{-2(2)}$

$= 0,0046$

$P[X > 10] = 1 - P[X \le 10] = 1 - (1 - \frac{1}{4} e^{-2(10)})$

$= 5.15 \cdot 10^{-10}$

4.14,

$P[X < -1] = 0$

$P[X \le -1] = \frac{2}{10}$

$P[-1 < X < -0,75] = P[X \le -0,75] - P[X \le -1]$

$= \frac{2}{10} - \frac{2}{10} = 0$

$P[-0,5 \le X \le 0,5] = P[X \le 0,5] - P[X \le -0,5]$

$= \frac{8}{10} - \frac{2}{10} = \frac{6}{10}$

$P[|X - 0,5| < 0,5] = P[\{X < 1\} \cup \{X > 0\}]$

$= 1 - \frac{6}{10} = \frac{4}{10}$

4.17, $1 = C \int_{-1}^{1} (1 - x^2) 2x = C \left[x \Big|_{-1}^{1} - \frac{x^3}{3} \Big|_{-1}^{1} \right]$

$= C \left[2 - \frac{1}{3} \cdot 2 \right] = \frac{4}{3} C$

$\Rightarrow C = \frac{3}{4}$

$f_X(x) = \frac{3}{4} (1 - x^2) \qquad -1 \le x \le 1$

$F_X(x) = \frac{3}{4} \int_{-1}^{x} (1 - y^2) dy = \frac{3}{4} \left[y \Big|_{-1}^{x} - \frac{y^3}{3} \Big|_{-1}^{x} \right]$

HẢI TIẾN

$= \frac{3}{4}\left[(x+1) - \frac{1}{3}(x^3+1) \right]$

$P[x=0] = F_x(0) = 0$

$P[0 < x < 0,5] = \frac{3}{4}\left[\left(\frac{1}{2}+1\right) - \frac{1}{3}\left(\frac{1}{8}+1\right) \right]$

$= \frac{3}{4}\left[1 - \frac{1}{3} \right] = \frac{11}{32}$

$P\left[\left| x - \frac{1}{2} \right| < \frac{1}{4} \right] = P\left[\frac{1}{4} < x < \frac{3}{4} \right]$

$= \frac{3}{4}\left[\left(\frac{3}{4}+1\right) - \frac{1}{3}\left(\left(\frac{3}{4}\right)^3 + 1 \right) \right]$

$- \frac{3}{4}\left[\left(\left(\frac{1}{4}+1\right) - \frac{1}{3}\left[\left(\frac{1}{4}\right)^3 + 1 \right] \right) \right]$

$= 0,2734$

4.19, $F_R(r) = \left(\frac{r}{2}\right)^2 \qquad 0 \le r \le 2$

a, $f_R(r) = \frac{d}{du} F_R(r) = 2\left(\frac{r}{2}\right)\left(\frac{1}{2}\right) = \frac{u}{2}$

b, $P\left[R > \frac{1}{4} \right] = \int_{1/4}^{2} \frac{r}{2}\, dr = \frac{1}{2} \frac{r^2}{2} \Big|_{1/4}^{2} = \frac{63}{64}$

4.20, $F_z(z) = 2 \cdot \frac{z}{b} - \frac{z^2}{b^2} \qquad (4.7)$

a, $f_z(z) = \frac{d}{dy} F_z(z) = \frac{2}{b} - \frac{2z}{b^2} = \frac{2}{b}\left(1 - \frac{z}{b}\right)$

b, $P\left[z > \frac{b}{3} \right] = \int_{b/3}^{b} \frac{2}{b}\left(1 - \frac{z}{b}\right) dz = \frac{2}{b}\left[z \Big|_{b/3}^{b} - \frac{1}{b} \frac{z^2}{2} \right.$

$= \frac{4}{9}$

No. ....
Date ....

4.30, a, $F_X(x|c) = \dfrac{P[\{X \le x\} \cap \{X > 0\}]}{P[X > 0]} = \dfrac{P[0 < X \le x]}{P[X > 0]}$

$$= \begin{cases} 0 & x \le 0 \\ \dfrac{F_X(x) - F_X(0)}{1 - F_X(0)} & x \ge 0 \end{cases}$$

$$= \begin{cases} 0 \\ \dfrac{-\frac{1}{4}e^{-2x} + \frac{1}{4}}{1 - (1 - \frac{1}{4})} = 1 - e^{-2x} \end{cases}$$

b, $F_X(x|c) = \dfrac{P[\{X \le x\} \cap \{X = 0\}]}{P[X = 0]}$

$$= \begin{cases} 0 & , X < 0 \\ 1 & , X \ge 0 \end{cases}$$

4.33, a, $F_X(x|X > T) = \dfrac{P[\{X \le x\} \cap \{X > t\}]}{P[X > t]}$

$$= \begin{cases} 0 & x < t \\ \dfrac{F_X(x) - F_X(t)}{1 - F_X(t)} & x \ge t \end{cases}$$

$$= \begin{cases} 0 \\ \dfrac{(1 - e^{-\lambda x}) - (1 - e^{-\lambda t})}{1 - (1 - e^{-\lambda t})} \end{cases}$$

$$= \begin{cases} 0 \\ \dfrac{e^{-\lambda x} - e^{-\lambda t}}{e^{-\lambda t}} \end{cases}$$

b, $f_X(x|X > t) = \dfrac{f_X(x)}{1 - F_X(t)} = \dfrac{\lambda e^{-\lambda x}}{e^{-\lambda t}} = \lambda e^{-\lambda(x-t)}$

c, $P = [X > t + x | X > t]$ $\quad x > 0$

$$= \frac{P[\{X > t + x\} \cap \{X > t\}]}{P[X > t]}$$

$$= \frac{1 - F_X(t+x)}{1 - F_X(t)}$$

$$= \frac{1 - (1 - e^{-\lambda(t+x)})}{1 - (1 - e^{-\lambda t})} = e^{-\lambda x} = P[X > x]$$

4.35) a, $F_X(x \mid a \le X \le b) = \dfrac{P[\{X \le x\} \cap \{a \le X \le b\}]}{P[a \le X \le b]}$

$$\{\{X \le x\} \cap \{a \le X \le b\} = \begin{cases} 0 & x < a \\ \{a \le X \le x & a \le x \le b \\ \{a \le X \le b & x > b \end{cases}$$

$$F_X(x \mid a \le X \le b) = \begin{cases} \dfrac{P[\phi]}{P[a \le X \le b]} = 0, & x < a \\ \dfrac{P[a < X < x]}{P[a \le X \le b]} = \dfrac{F_X(x) - F_X(a-1)}{F_X(b) - F_X(a-1)} \\ \dfrac{P[a < X < b]}{P[a \le X \le b]} = 1, & x > b \end{cases}$$

b, $f_X(x \mid a \le X \le b) = \dfrac{d}{dx} F_X(x \mid a \le X \le b)$

$$= \begin{cases} 0 & x < a \\ \dfrac{f_X(x)}{F_X(b) - F_X(a)} & a \le x \le b \\ 0 & x > b \end{cases}$$

4.36, $F_R(r \mid R > 1) = \dfrac{P[R \le r, R > 1]}{P[R > 1]}$ $\quad 1 \le r \le 2$

$$= \frac{P[1 \le R \le r]}{P[R > 1]} = \frac{\left(\frac{r^2}{2}\right)^2 - \left(\frac{1}{2}\right)^2}{1 - \left(\frac{1}{2}\right)^2} = \frac{r^2 - 1}{3}$$

$$f_Y(r \mid R > 1) = \frac{d}{dr} \frac{1}{3}(r^2 - 1) = \frac{2}{3} r \qquad 1 \le r \le 2$$

4.38) a, $F_Y(x) = F_Y(x \mid B_0) P[B_0] + F_Y(x \mid B_1) P[B_1]$

$= P[Y \le x \mid X = -1](1-p) + P[Y \le x \mid X = 1] p$

$= P[X + N \le x \mid X = -1](1-p) + P[X+N \le x \mid X = 1] p$

$= P[N \le x + 1](1-p) + P[N \le x - 1] p$

$= F_N(x+1)(1-p) + F_N(x-1) p$

$$f_Y(x) = \frac{d}{dx} F_Y(x) = (1-p) f_N(x+1) + p f_N(x-1)$$

$$f_Y(x \mid B_0) = f_N(x+1) = \frac{\alpha}{2} e^{-\alpha|x+1|}$$

$$f_Y(x \mid B_1) = f_N(x-1) = \frac{\alpha}{2} e^{-\alpha|x+1|}$$

b) $$f_Y(x) = \frac{1}{2}\left[\frac{\alpha}{2} e^{-\alpha|x+1|} + \frac{\alpha}{2} e^{-\alpha|x-1|}\right]$$

b, $P[Y < 0 \mid B_1] = P[X + N < 0 \mid X = 1] = P[N < -1]$

$= \frac{\alpha}{2} e^{-\alpha|-1|} = \frac{\alpha}{2} e^{-\alpha}$

$P[Y \ge 0 \mid B_0] = P[X + N \ge 0 \mid X = -1] = P[N \ge 1]$

$= \frac{\alpha}{2} e^{-\alpha}$

c, $P_E = P[Y < 0 \mid B_1] P[B_1] + P[Y \ge 0 \mid B_0] P[B_0]$

$= 0,5 \frac{\alpha}{2} \cdot e^{-\alpha} + 0,5 \frac{\alpha}{2} e^{-\alpha} = \frac{\alpha}{2} e^{-\alpha}$

4.41, $f_R(r) = \frac{r}{2} \qquad 0 \le r \le 2$

$$E[R] = \int_0^2 r\left(\frac{r}{2}\right) dr = \frac{4}{3}$$

$$E[R^2] = \int_0^2 r^2 \frac{r}{2} dr = 2$$

$$VAR[R] = E[R^2] - E[R]^2 = \frac{2}{9}$$

IẾN

4.42, $f_Z(z) = \frac{2}{b}\left(1 - \frac{z}{b}\right) \quad 0 \le z \le b$

$$E[Z] = \frac{2}{b}\int_0^b z\left(1-\frac{z}{b}\right)dz = \frac{2}{b}\left[\frac{z^2}{2} - \frac{1}{b}\frac{z^3}{3}\right]\Big|_0^b$$

$$= \frac{2}{b}\left[\frac{b^2}{2} - \frac{1}{b}\frac{b^3}{3}\right] = b\left[1 - \frac{2}{3}\right] = \frac{b}{3}$$

$$E[Z^2] = \frac{2}{b}\int_0^b z^2\left(1-\frac{z}{b}\right)dz = \frac{2}{b}\left[\frac{z^3}{3} - \frac{1}{b}\frac{z^4}{4}\right]_0^b$$

$$= 2b^2\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{b^2}{6}$$

$$VAR[Z] = E[Z^2] - E[Z]^2 = \frac{b^2}{6} - \frac{b^2}{9} = \frac{b^2}{18}$$

4.54, a, $E[Y] = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$

$$= -a\int_{-\infty}^{a} f_X(x)\,dx + \int_{-a}^{a} x f(x)\,dx + a(1-F_X(a^-))$$

$$E[Y^2] = a^2 F_X(-a) + \int_{-a}^{a} x^2 f_X(x)\,dx + a^2(1-F_X(a^-))$$

$$VAR[Y] = E[Y^2] - E[Y]^2$$

b, $E[Y] = -(1)\,P[Y \le -1] + (1)\,P[Y \ge 1] + \int_{-1}^{1} x\frac{1}{2}e^{-|x|}dx$

$$= -\frac{1}{2}e^{-1} + \frac{1}{2}e^{-1} + 0 = 0$$

$$VAR[Y] = E[Y^2] = (-1)^2 P[Y \le -1] + (1)^2 P[Y \ge 1] + \int_{-1}^{1} x^2 \frac{1}{2} e^{-|x|}dx$$

$$= e^{-1} + 2\cdot\frac{1}{2}\int_0^1 x^2 e^{-x}\,dx$$

$$= e^{-1} + 5e^{-1} - 2 = 6e^{-1} - 2$$

4.60,

4.61, a, $P[X \le d] = F_X(d) = 1 - e^{\lambda d}$

$P[kd \le X \le (k+1)d] = F_X((k+1)d) - F_X(kd)$

$= e^{-\lambda kd} - e^{-\lambda(k+1)d}$

$= e^{-\lambda kd}(1 - e^{-\lambda d})$

$P[X > kd] = 1 - P[X \le kd] = 1 - F_X(kd) = e^{-\lambda kd}$

4.63, a, $P[X > 4] = 1 - F_X(4) = 1 - \Phi\frac{4-5}{4} = 0,598$

$P[X \ge 7] = 1 - F_X(7) = 1 - \Phi\left(\frac{7-5}{4}\right) = 0,308$

$P[6,72 < X < 10,16] = \Phi\left(\frac{10,16-5}{4} - \frac{6,72-5}{4}\right) = 0,235$

$P[2 < X < 7] = \Phi\left(\frac{7-5}{4} - \frac{2-5}{4}\right) = 0,465$

b, $P[X < a] = 0,8869$

$\Phi\frac{a-5}{4} = 0,8869 = 1 - Q(x)$

$Q(x) = 0,1131 \Rightarrow x = 1,2 = \frac{a-5}{4} \Rightarrow a = 9,8$

c, $P[X > b] = 1 - \Phi\frac{b-5}{4} = 0,1131$

$Q(x) = 0,1131 \Rightarrow x = 1,2 = \frac{b-5}{4} \Rightarrow b = 9,8$

4.76, $P[Y \le y] = P\left[\frac{1}{X+1} \le y\right]$

$= P\left[\frac{1}{y} \le X+1\right]$

$= P\left[X \ge \frac{1}{y} - 1\right]$

$= 1 - \left(\frac{1}{y} - 1\right) = 2 - \frac{1}{y}$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{y^2}$$

4.77, $Z = (X-\alpha)^+ = \begin{cases} 0 & x \le \alpha \\ x-\alpha & x > \alpha \end{cases}$

$$P[Z \le z] = \begin{cases} 0 & z < 0 \\ P[X \le r] = 1 - e^{-\alpha^2/2\alpha^2} = 1 - e^{-1/2}, & z = 0 \\ P[X = \alpha \le z] = P[X \le z + \alpha] \\ \quad = 1 - e^{-(z+\alpha)^2/2\alpha^2}, & z > 0 \end{cases}$$

$$f_Z(z) = \frac{d}{dz} F_Z(z) = (1 - e^{-1/2})\,\delta(z) + \frac{d}{dz} F_X(z+\alpha)$$

$$= (1 - e^{-1/2})\,\delta(z) + f_X(z+\alpha)$$

$$= (1 - e^{-1/2})\,\delta(z) + \frac{z+\alpha}{\alpha^2} \cdot e^{-(z+\alpha)^2/2\alpha^2}$$

4.80, $Y = 2X + 3$

$$F_Y(y) = P[2X + 3 \le y] = P\left[X \le \frac{y-3}{2}\right]$$

$$= F_X\left(\frac{y-3}{2}\right)$$

$$\Rightarrow f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} F_X\left(\frac{y-3}{2}\right)$$

$$= \frac{1}{2} f_X\left(\frac{y-3}{2}\right)$$

$$f_X(x) = \frac{3}{4}(1 - x^2)$$

$$\Rightarrow f_Y(y) = \frac{3}{8}\left(1 - \left(\frac{y-3}{2}\right)^2\right)$$

4.90, $X = \pm\sqrt{\frac{P}{R}}$ $\quad \frac{dx}{dp} = \pm \frac{1}{2} \frac{P^{\frac{1}{2}-1}}{\sqrt{R}} = \pm \frac{1}{2\sqrt{RP}}$

$$f_P(p) = [f_X(x) + f_X(-x)] \left|\frac{dx}{dp}\right|$$

IẾN

$$= \left[f_X\left(\sqrt{\frac{P}{R}}\right) + f_X\left(-\sqrt{\frac{P}{R}}\right)\right] \frac{1}{2\sqrt{RP}}$$

$$= \left[\frac{1}{2\sqrt{2\pi}} \left(e^{-\left(\sqrt{\frac{P}{R}}-1\right)^2/2\cdot 2} + e^{-\left(-\sqrt{\frac{P}{R}}-1\right)^2/2\cdot 2}\right)\right] \frac{1}{2\sqrt{RP}}$$

$$= \frac{1}{2\sqrt{2\pi}} \cdot \frac{P}{2\sqrt{RP}} \left(e^{-(\sqrt{P}-\sqrt{R})^2/2\cdot(2R)} + e^{-(\sqrt{P}-(-\sqrt{R}))^2/2\cdot 2R} \right)$$

4.91, a, $y \le 0$: $P[Y \le y] = 0$

$y > 0$ $P[Y \le y] = P[e^X \le y] = F_X(\ln y)$

$y > 0$: $f_Y(y) = \frac{d}{dy} F_Y(y) = F'_X(\ln y) \frac{d}{dy} \ln y$

$= y\, f_X(\ln y)$

b, $f_Y(y) = \begin{cases} 0, & y \le 0 \\ \dfrac{e^{-(\ln y - m)^2/2\sigma^2}}{y\sqrt{2\pi}\,\sigma}, & y > 0 \end{cases}$

4.102, $\Phi_X(w) = \int_{-\infty}^{\infty} f_X(x)\, e^{jwx}\, dx$

$= \int_{-b}^{b} \frac{1}{b-a} e^{jwx}\, dx$

$= \frac{e^{jwb} - e^{-jwb}}{jw(b-(-b))} = \frac{e^{jwb} - e^{-jwb}}{2jwb}$

$E[X] = \frac{1}{j} \frac{d\Phi_X(w)}{dw}\Big|_{w=0}$

$= -\frac{1}{b+b} \cdot \left[-\frac{1}{2} b^2 + \frac{1}{2} b^2 \right] = 0$

$E[X^2] = \frac{1}{j^2} \cdot \frac{d^2\Phi_X(w)}{dw^2}\Big|_{w=0}$

TIẾN

$$= -\frac{1}{j(b-a)}\left[-\frac{1}{3}jb^3 + \frac{1}{3}j\,a^3\right]_{a=-b}$$

$$= \frac{b^2}{3}$$

$$VAR[X] = E[X^2] - E^2[X] = \frac{b^2}{3}$$

$\boxed{\text{Chương 5:}}$

5.3,

a, $S_{XY} = \{(-1,-1), (-1,0), (-1,1), (1,-1), (1,0), (1,1)\}$

b, $P[X=1, Y=-1] = P[Y=-1 \mid X=1] \cdot P[X=1] = 3/4\,p$

$P[X=-1, Y=-1] = 1/4\,(1-p-p_e)$

$P[X=1, Y=0] = P[Y=0 \mid X=1]\,P[X=1] = 3/4\,p_e$

$P[X=-1, Y=0] = 1/4\,p_e$

$P[X=1, Y=1] = P[Y=1 \mid X=1]\,P[X=1] = \frac{3}{4}(1-p-p_e)$

$P[X=-1, Y=1] = 1/4\,p$

c, $P[X \neq Y] = \frac{1}{4}p_e + \frac{1}{4}p + \frac{3}{4}p_e + \frac{3}{4}p$

$= p_e + p$

d, $P[Y=0] = \frac{3}{4}p_e + \frac{1}{4}p_e = p_e$

5.13, a) $S_{XY} = \{ (i,j) : 0 \le i \le 100, 0 \le j \le 100 \}$

b) $P_{XY}(x,y) = \binom{100}{x} (0{,}05)^x \cdot (0{,}95)^{100-x} \cdot \binom{100}{y} (0{,}05)^y \cdot 0{,}95^{(100-y)}$

c) $P[X = x] = \binom{100}{x} \, 0{,}05^x \, 0{,}95^{100-x}$

$P[Y = y] = \binom{100}{y} \, 0{,}05^y \, 0{,}95^{100-y}$

5.25 a) $F_{X,Y}(x,y) = \int_0^x \int_0^y \frac{1}{2} e^{-x/2} \, 2y e^{-y^2} \, dx dy$

$= (1 - e^{-x/2})(1 - e^{-y^2})$

b) $P[X > Y] = \int_0^\infty \int_0^x 2y e^{-y^2} dy \, \frac{1}{2} e^{-x/2} dx$

$= \int_0^\infty (1 - e^{-x^2}) \frac{1}{2} e^{-x/2} dx$

$= 1 - \frac{1}{2} \int_0^\infty e^{-(x^2 + x/2)} dx$

$= 1 - \frac{1}{2} e^{1/16} \int_0^\infty e^{-(x + \frac{1}{4})^2} dx$

$= 1 - \frac{\sqrt{\pi}}{2} \cdot e^{1/16}$

c) $F_X(x) = \lim_{y \to \infty} F_{XY}(x,y) = 1 - e^{-x/2}$

$f_X(x) = \frac{1}{2} e^{-x/2}$

$F_Y(y) = 1 - e^{-y^2}$

$f_Y(y) = 2y e^{-y^2}$

5.28, a) i) $K\pi = 1 \Rightarrow K = 1/\pi$

ii) $K \cdot \sqrt{2} \cdot \sqrt{2} = 1 \Rightarrow K = \frac{1}{2}$

iii) $K \cdot 1^2/2 = 1 \Rightarrow K = 2$

b, i, $f_X(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx = \dfrac{2\sqrt{1-x^2}}{\pi}$

$f_Y(y) = \dfrac{2\sqrt{1-y^2}}{\pi}$

ii, $f_X(x) = \int_{|x|-1}^{1-|x|} \dfrac{dy}{2} = 1-|x|$

$f_Y(y) = 1-|y|$

5.48, a, $P[X^2 < \frac{1}{2}, |Y| < \frac{1}{2}] = P[X^2 < \frac{1}{2}]\, P[|Y| < \frac{1}{2}]$
$= 1/\sqrt{2} \cdot 1/2 = 1/2\sqrt{2}$

b, $P[4X < 1, Y < 0] = P[X < \frac{1}{4}]\, P[Y < 0] = 1/4 \cdot 0 = 0$

c, $f(x,y) = f(x) \cdot f(y) = 1$

$P[XY < \frac{1}{2}] = \dfrac{1}{2} + \int_{1/2}^{1} \int_{0}^{1/2x} 1\, dy\, dx$

$= \dfrac{1}{2} + \int_{1/2}^{1} \dfrac{1}{2x}\, dx$

$= 0,85$

5.56, a, $E[(X+Y)^2] = E[X^2 + 2XY + Y^2] = E[X^2] + 2E[XY] + E[Y^2]$

b, $VAR[X+Y] = E[(X+Y)^2] - E[X+Y]^2$
$= E[X^2] + 2E[XY] + E[Y^2] - E[X]^2$
$- 2E[X]E[Y] - E[Y]^2$
$= VAR[X] + VAR[Y] + 2[E[XY] - E[X]E[Y]]$

5.72, $E[XY] = \int_0^{2\pi} \cos\dfrac{\theta}{4} \sin\dfrac{\theta}{4} \dfrac{d\theta}{2\pi}$

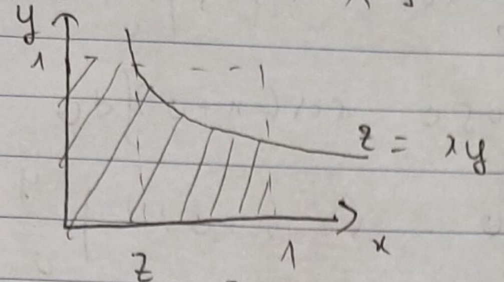
$= \dfrac{1}{4\pi} \int_0^{2\pi} \sin\dfrac{\theta}{2}\, d\theta = \dfrac{1}{\pi}$

5.95, $Z = XY$ $f_{XY}(x,y) = 1$

$F_Z(z) = P[Z \le z]$

$= P[XY \le z]$

$= P[Y \le z/x]$



$= z + \int_z^1 \int_0^{z/x} dy\,dx$

$= z + \int_z^1 \frac{z}{x}\,dx$

$= z - z\ln z.$

$f_Z(z) = \frac{d}{dz} F_Z(z) = 1 - \ln z - 1$

$= \begin{cases} -\ln z, & 0 \le z \le 1 \\ 0, & \ldots \end{cases}$

5.129, a, $\int_0^{\pi/2}\int_0^{\pi/2} c\sin(x+y)\,dx\,dy = 1$

$\Rightarrow c\int_0^{\pi/2} \left[\left[-\cos(x+y)\right]\Big|_0^{\pi/2}\right] dy = 1$

$\Rightarrow c\int_0^{\pi/2} \left(\cos y - \cos\left(\frac{\pi}{2} + y\right)\right) dy = 1$

$c\int_0^{\pi/2} (\cos y + \sin y)\,dy = 1$

$2c\sin y\Big|_0^{\pi/2} = 1 \Rightarrow c = \frac{1}{2}$

b, $F_{X,Y}(x,y) = \int_0^y\int_0^x \frac{1}{2}\sin(u+v)\,du\,dv$

No. ....
Date ....

$$= \int_0^y \left[-\frac{1}{2} \cos(u+v) \right] \Big|_0^x dv$$

$$= \frac{1}{2} \int_0^y \left[\cos v - \cos(x+v) \right] dv$$

$$= \frac{1}{2} \left(\sin v - \sin(x+v) \right) \Big|_0^y$$

$$= \frac{1}{2} \left(\sin y - \sin(x+y) + \sin x \right)$$

C, $f_X(x) = \int_0^{\pi/2} \frac{1}{2} \sin(x+y')\, dy'$

$$= \frac{1}{2} \left(-\cos(x+y') \right) \Big|_0^{\pi/2}$$

$$= \frac{1}{2} \left(\cos x + \sin x \right)$$

$$f_Y(y) = \frac{1}{2} \left(\cos y + \sin y \right)$$