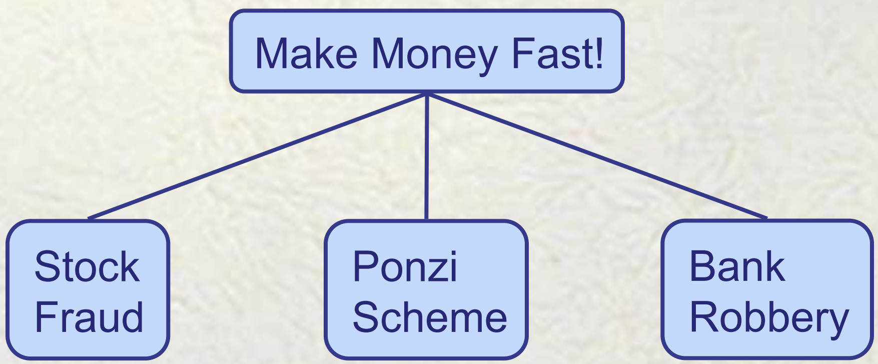
Data Structures and Algorithms

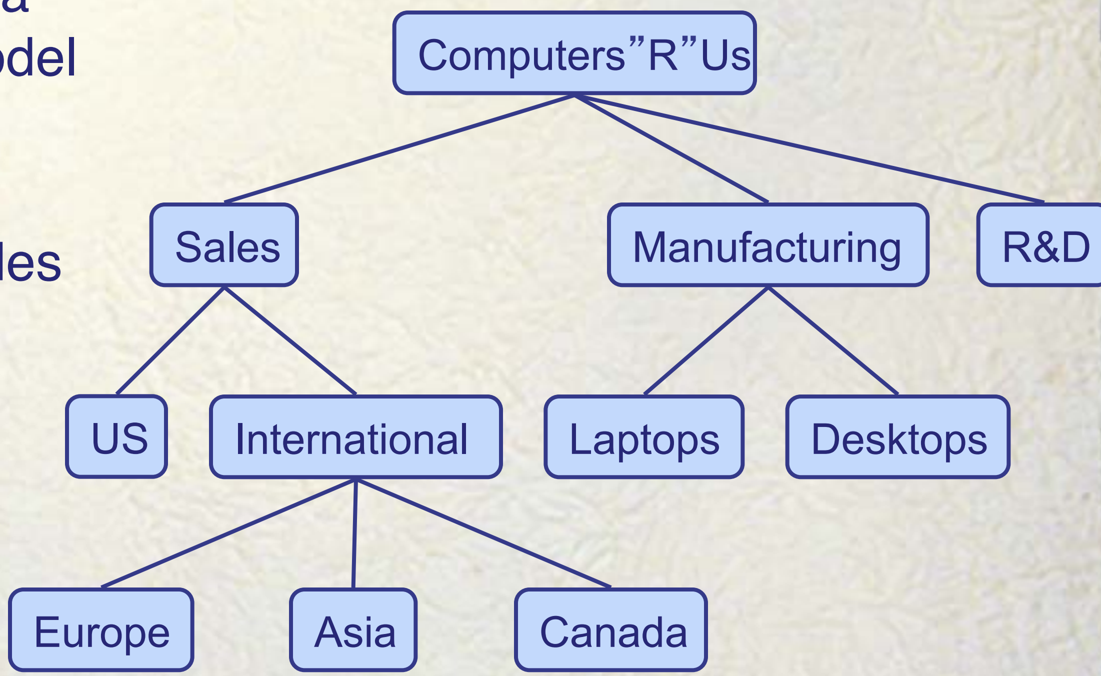
Trees

Tree Example

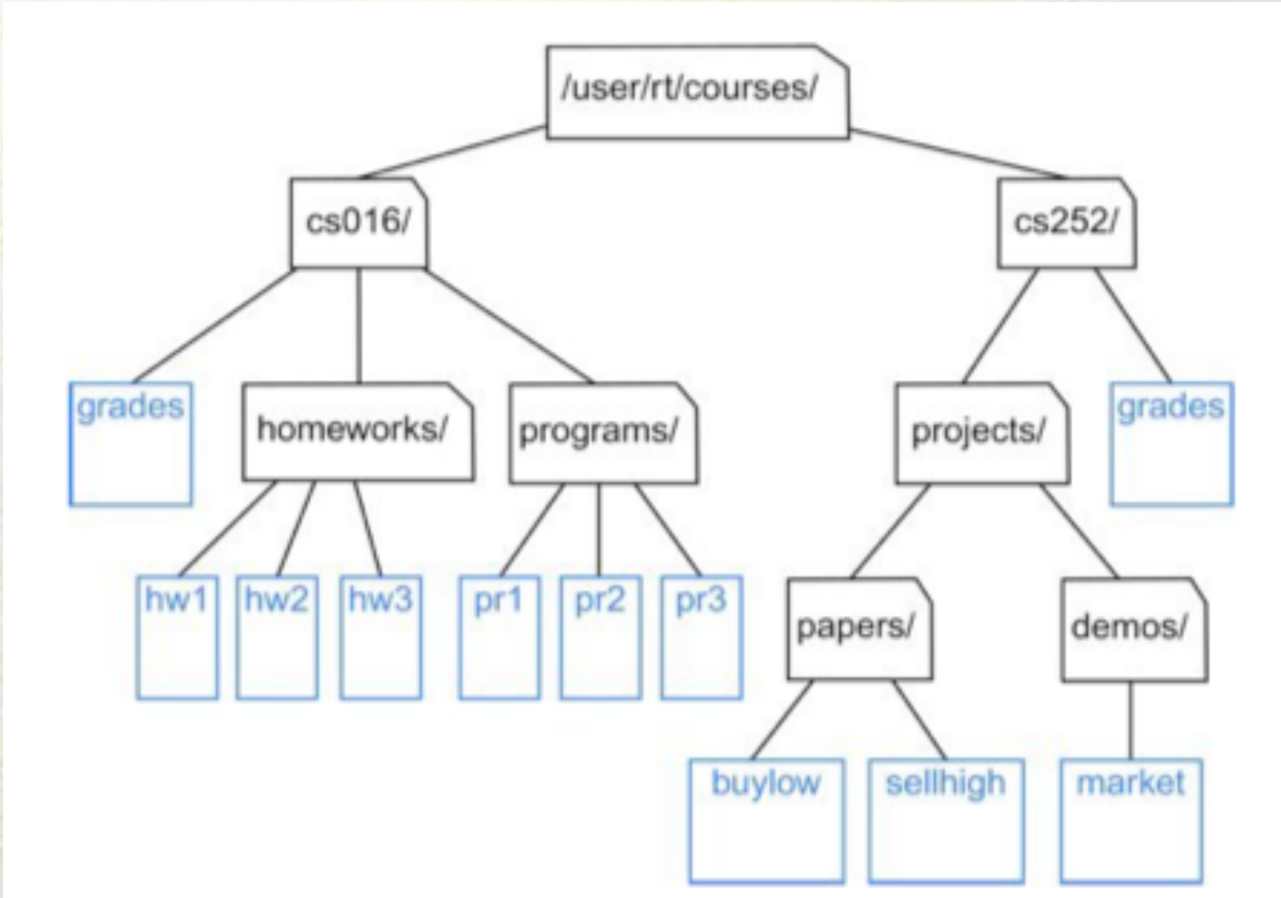


What is a Tree

- In computer science, a tree is an abstract model of a hierarchical structure
- A tree consists of nodes with a parent-child relation
- Applications:
  - Organization charts
  - File systems
  - Programming environments



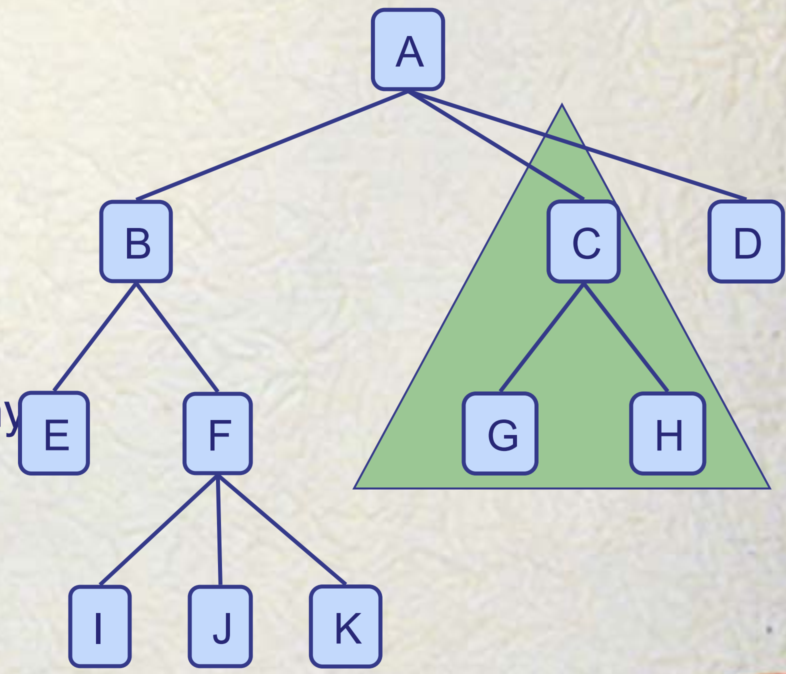
Tree: File system



Tree Terminology

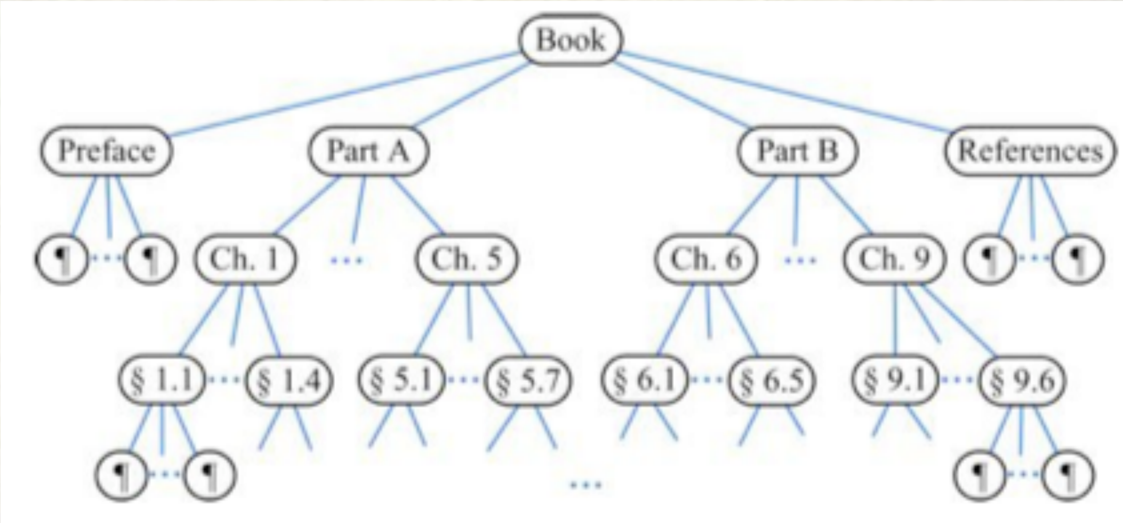
- Root: node without parent (A)
- Internal node: node with at least one child (A, B, C, F)
- External node (a.k.a. leaf): node without children (E, I, J, K, G, H, D)
- Ancestors of a node: parent, grandparent, grand-grandparent, etc.
- Descendant of a node: child, grandchild, grand-grandchild, etc.
- Depth of a node: number of ancestors
- Height of a tree: maximum depth of any node (3)
- Siblings: same parent.
- Edge: (u, v): u is the parent of v.
- Path

- Subtree: tree consisting of a node and its descendants



Ordered Tree

- Linear ordering for children of each node.
- Example: Book structure

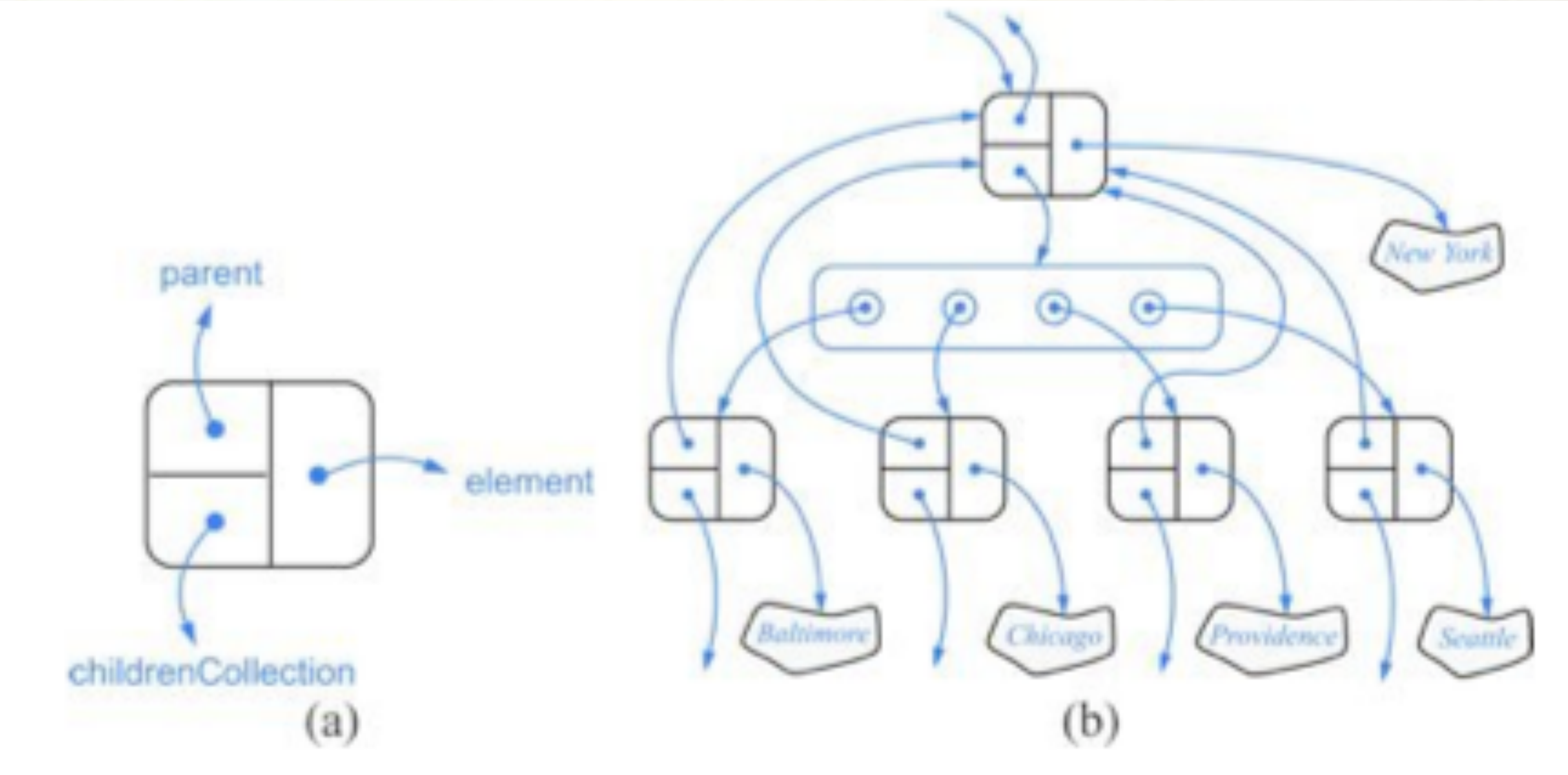


Tree ADT

- We use positions to abstract nodes
- Generic methods:
  - integer size()
  - boolean isEmpty()
  - Iterator elements()
  - Iterator positions()
- Accessor methods:
  - position root()
  - position parent(p)
  - positionIterator children(p)

- Query methods:
  - boolean isInternal(p)
  - boolean isExternal(p)
  - boolean isRoot(p)
- Update method:
  - object replace (p, o)
- Additional update methods may be defined by data structures implementing the Tree ADT

Tree Linked Structure



Depth

- Depth(v): number of ancestors of v.

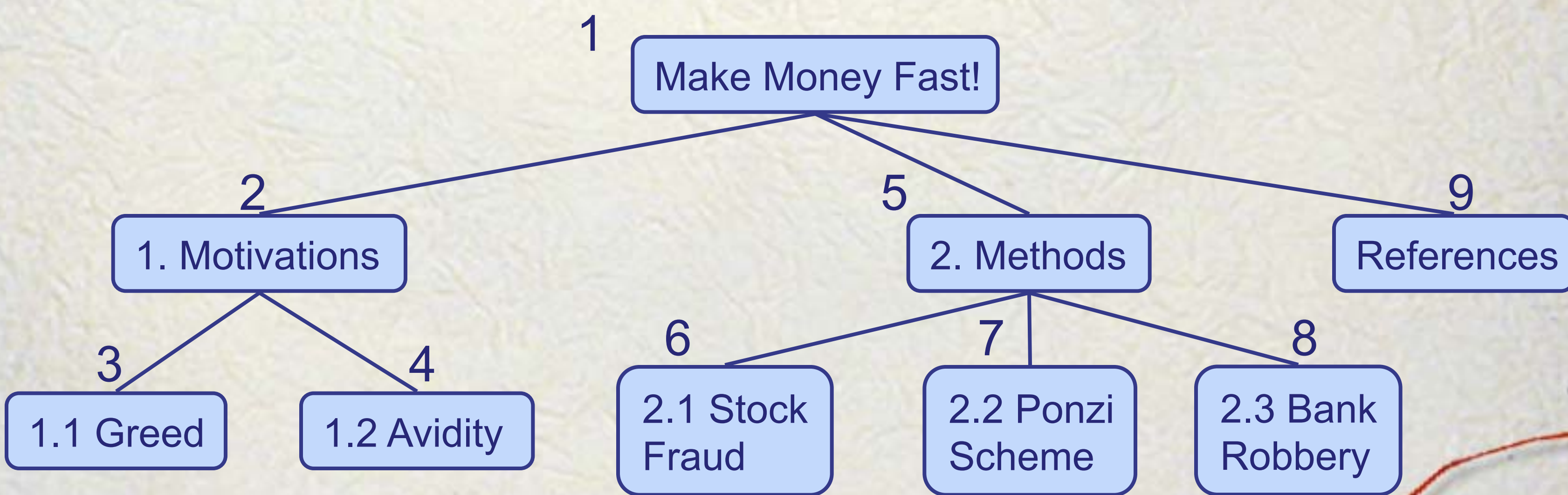
Algorithm depth(T, v):
  if v is the root of T **then**
    return 0
  else
    return $1 + \text{depth}(T, w)$, where w is the parent of v in T

```
public static <E> int depth (Tree<E> T, Position<E> v) {
  if (T.isRoot(v))
    return 0;
  else
    return 1 + depth(T, T.parent(v));
}
```

Preorder Traversal

- A traversal visits the nodes of a tree in a systematic manner
- In a preorder traversal, a node is visited before its descendants
- Application: print a structured document

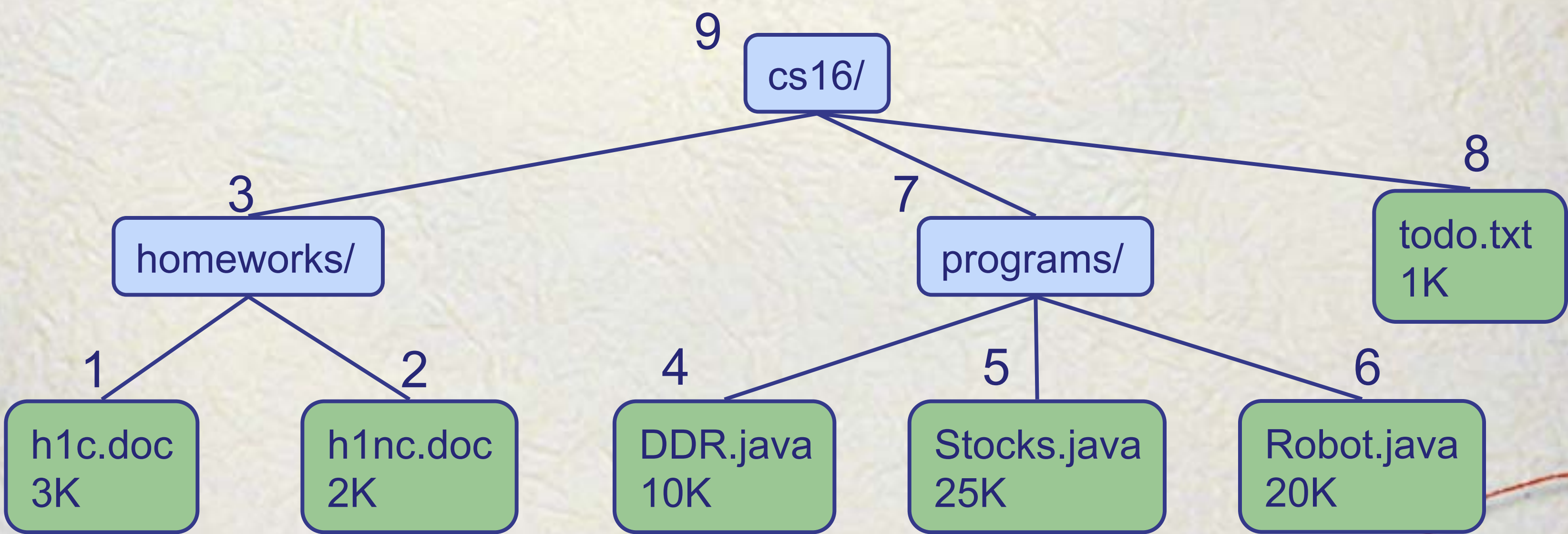
Algorithm *preOrder*(*v*)
    visit(*v*)
    for each child *w* of *v*
        preorder (*w*)



Postorder Traversal

- In a postorder traversal, a node is visited after its descendants
- Application: compute space used by files in a directory and its subdirectories

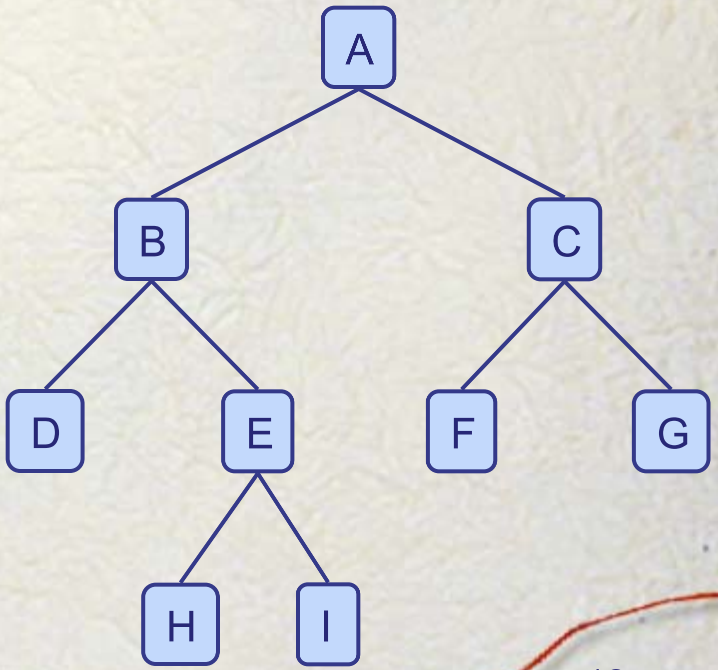
Algorithm *postOrder*(*v*)
  for each child *w* of *v*
    postOrder (*w*)
  visit(*v*)



Binary Trees

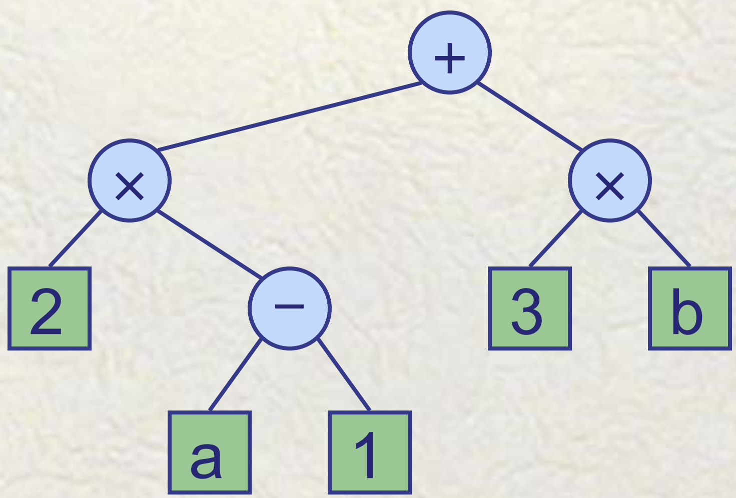
- A binary tree is a tree with the following properties:
  - Each internal node has at most two children (exactly two for **proper** binary trees)
  - The children of a node are an ordered pair
- We call the children of an internal node left child and right child
- Alternative recursive definition: a binary tree is either
  - a tree consisting of a single node, or
  - a tree whose root has an ordered pair of children, each of which is a binary tree

- Applications:
  - arithmetic expressions
  - decision processes
  - searching



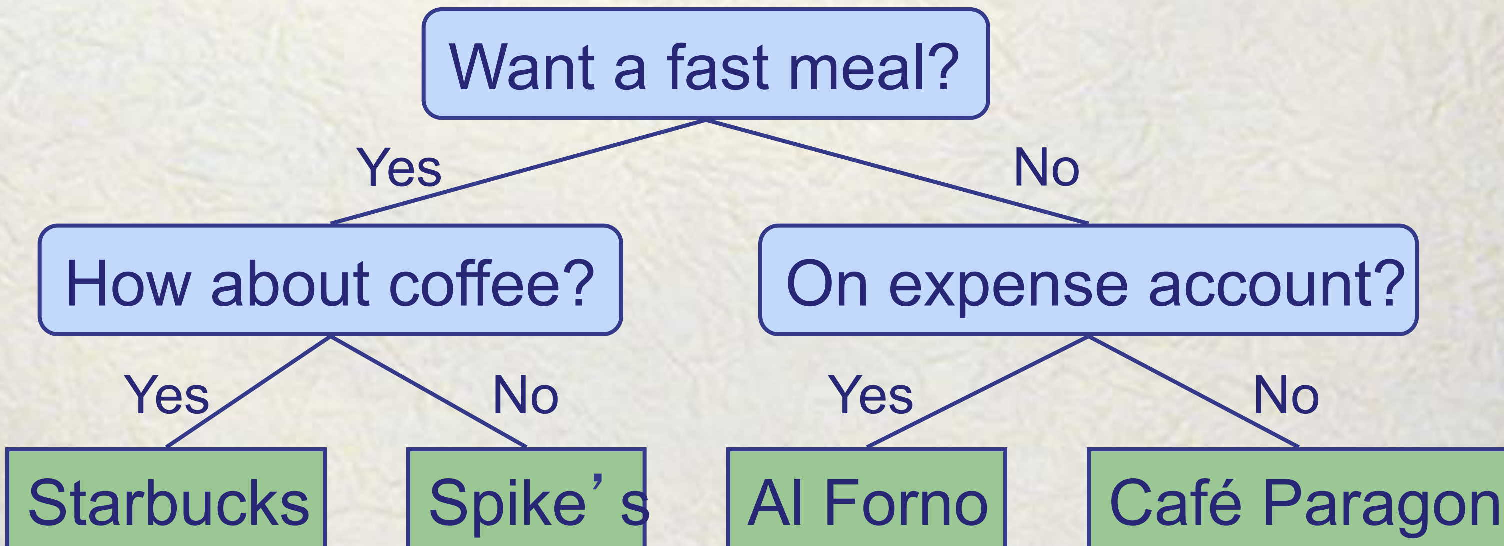
Arithmetic Expression Tree

- Binary tree associated with an arithmetic expression
  - internal nodes: operators
  - external nodes: operands
- Example: arithmetic expression tree for the expression $(2 \times (a - 1) + (3 \times b))$



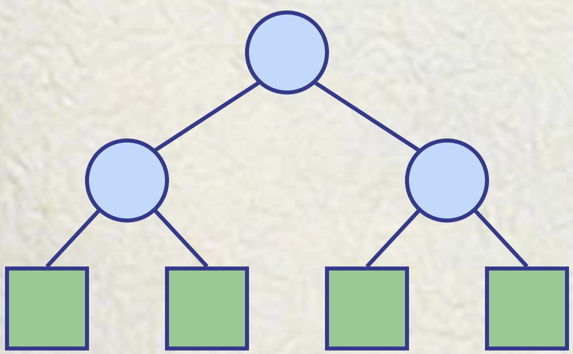
Decision Tree

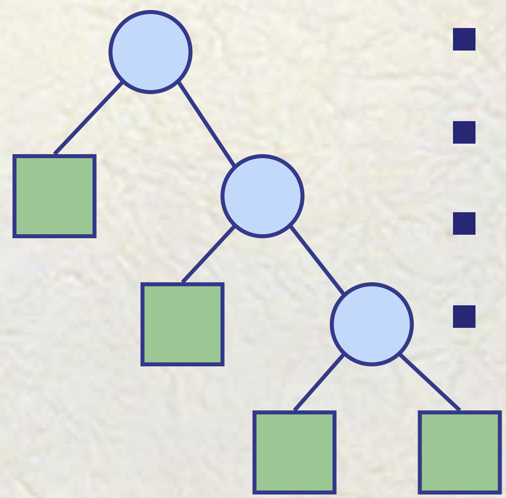
- Binary tree associated with a decision process
  - internal nodes: questions with yes/no answer
  - external nodes: decisions
- Example: dining decision



Properties of Proper Binary Trees

- Notation
  - n number of nodes
  - e number of external nodes
  - i number of internal nodes
  - h height





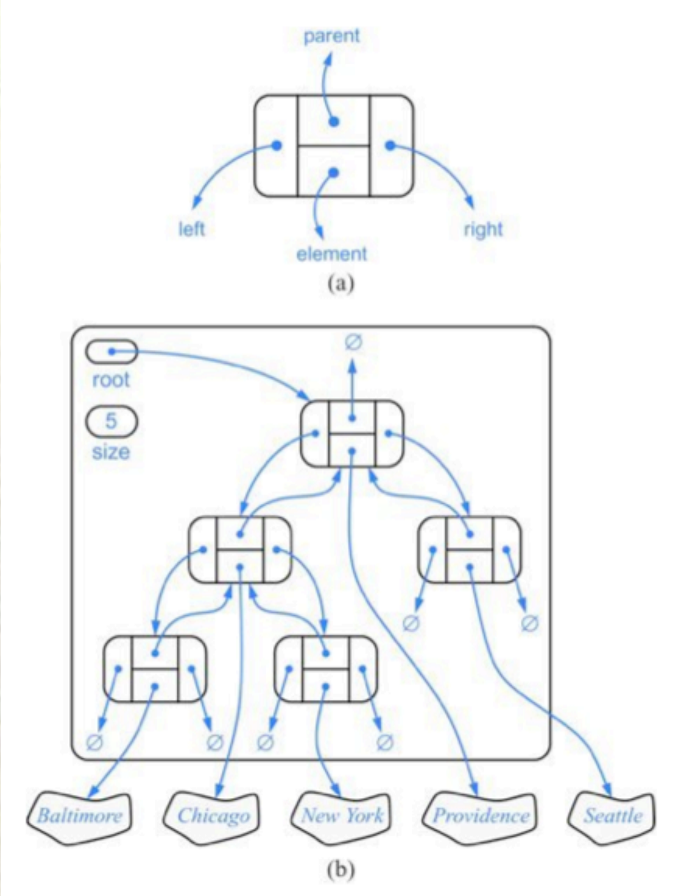
- Properties:
  - $e = i + 1$
  - $n = 2e - 1$
  - $h \leq i$
  - $h \leq (n - 1)/2$
  - $e \leq 2^h$
  - $h \geq \log_2 e$
  - $h \geq \log_2 (n + 1) - 1$

BinaryTree ADT

- The BinaryTree ADT extends the Tree ADT, i.e., it inherits all the methods of the Tree ADT
- Additional methods:
  - position left(p)
  - position right(p)
  - boolean hasLeft(p)
  - boolean hasRight(p)

- Update methods may be defined by data structures implementing the BinaryTree ADT

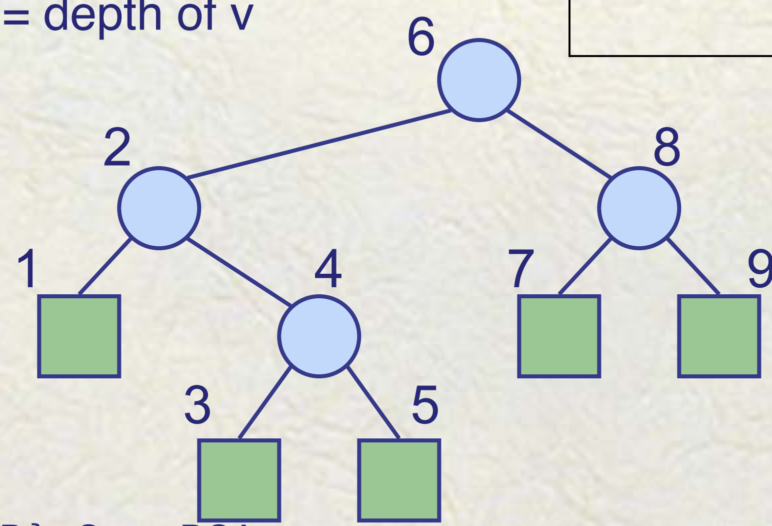
Linked Structure

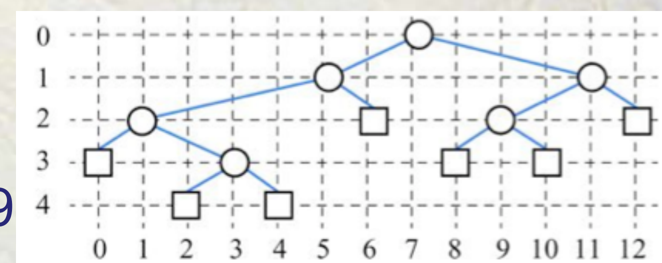


Inorder Traversal

- In an inorder traversal a node is visited after its left subtree and before its right subtree
- Application: draw a binary tree
  - x(v) = inorder rank of v
  - y(v) = depth of v

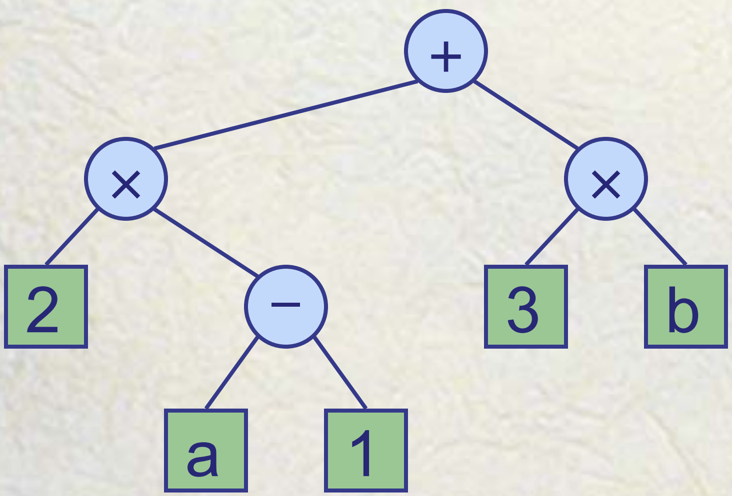
```
Algorithm inOrder(v)
    if hasLeft (v)
        inOrder (left (v))
    visit(v)
    if hasRight (v)
        inOrder (right (v))
```





Print Arithmetic Expressions

- Specialization of an inorder traversal
  - print "(" before traversing left subtree
  - print operand or operator when visiting node
  - print ")" after traversing right subtree



Algorithm *printExpression*(*v*)

    if *hasLeft* (*v*)

        print("(")

        printExpression(*left*(*v*))

    print(*v.element* ())

    if *hasRight* (*v*)

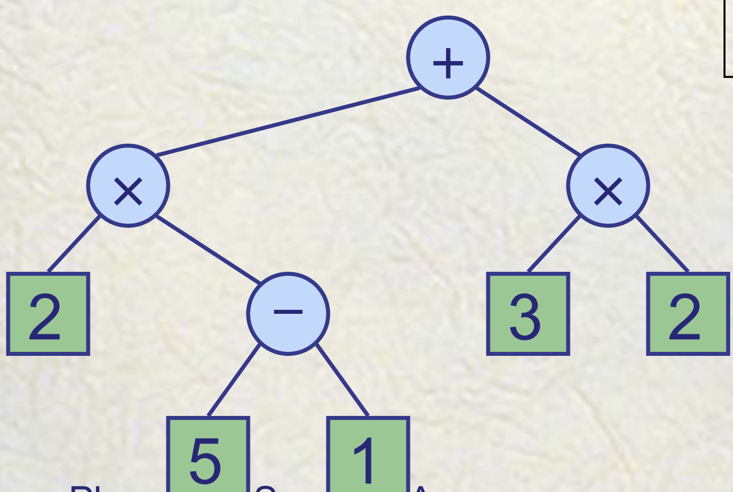
        printExpression(*right*(*v*))

        print (")")

$((2 \times (a - 1)) + (3 \times b))$

Evaluate Arithmetic Expressions

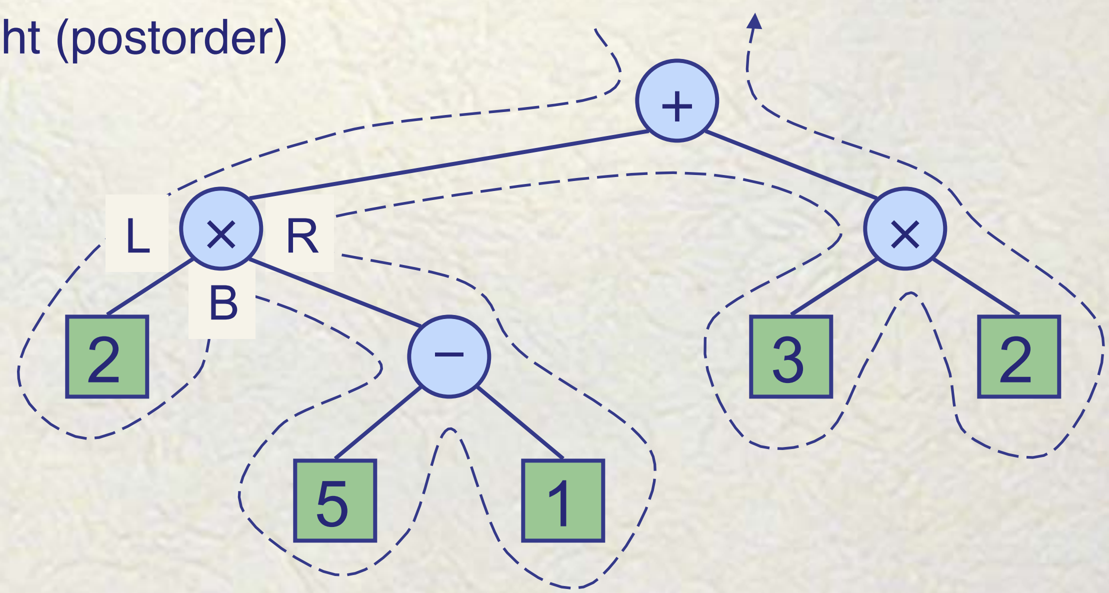
- Specialization of a postorder traversal
  - recursive method returning the value of a subtree
  - when visiting an internal node, combine the values of the subtrees



Algorithm *evalExpr*(v)
    if *isExternal* (v)
        return *v.element* ()
    else
        $x \leftarrow$ *evalExpr*(*leftChild* (v))
        $y \leftarrow$ *evalExpr*(*rightChild* (v))
        $\Diamond \leftarrow$ operator stored at v
        return $x \Diamond y$

Euler Tour Traversal

- Generic traversal of a binary tree
- Includes special cases for preorder, postorder and inorder traversals
- Walk around the tree and visit each node three times:
  - on the left (preorder)
  - from below (inorder)
  - on the right (postorder)



Template Method Pattern

- Generic algorithm that can be specialized by redefining certain steps
- Implemented by means of an abstract Java class
- Visit methods that can be redefined by subclasses
- Template method eulerTour
  - Recursively called on the left and right children
  - A Result object with fields leftResult, rightResult and finalResult keeps track of the output of the recursive calls to eulerTour

```
public abstract class EulerTour {
    protected BinaryTree tree;
    protected void visitExternal(Position p, Result r) { }
    protected void visitLeft(Position p, Result r) { }
    protected void visitBelow(Position p, Result r) { }
    protected void visitRight(Position p, Result r) { }
    protected Object eulerTour(Position p) {
        Result r = new Result();
        if tree.isExternal(p) { visitExternal(p, r); }
        else {
                visitLeft(p, r);
                r.leftResult = eulerTour(tree.left(p));
                visitBelow(p, r);
                r.rightResult = eulerTour(tree.right(p));
                visitRight(p, r);
                return r.finalResult;
        } …
```

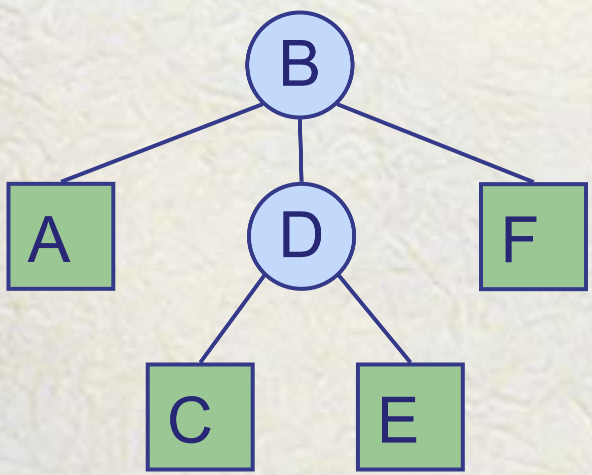
Specializations of EulerTour

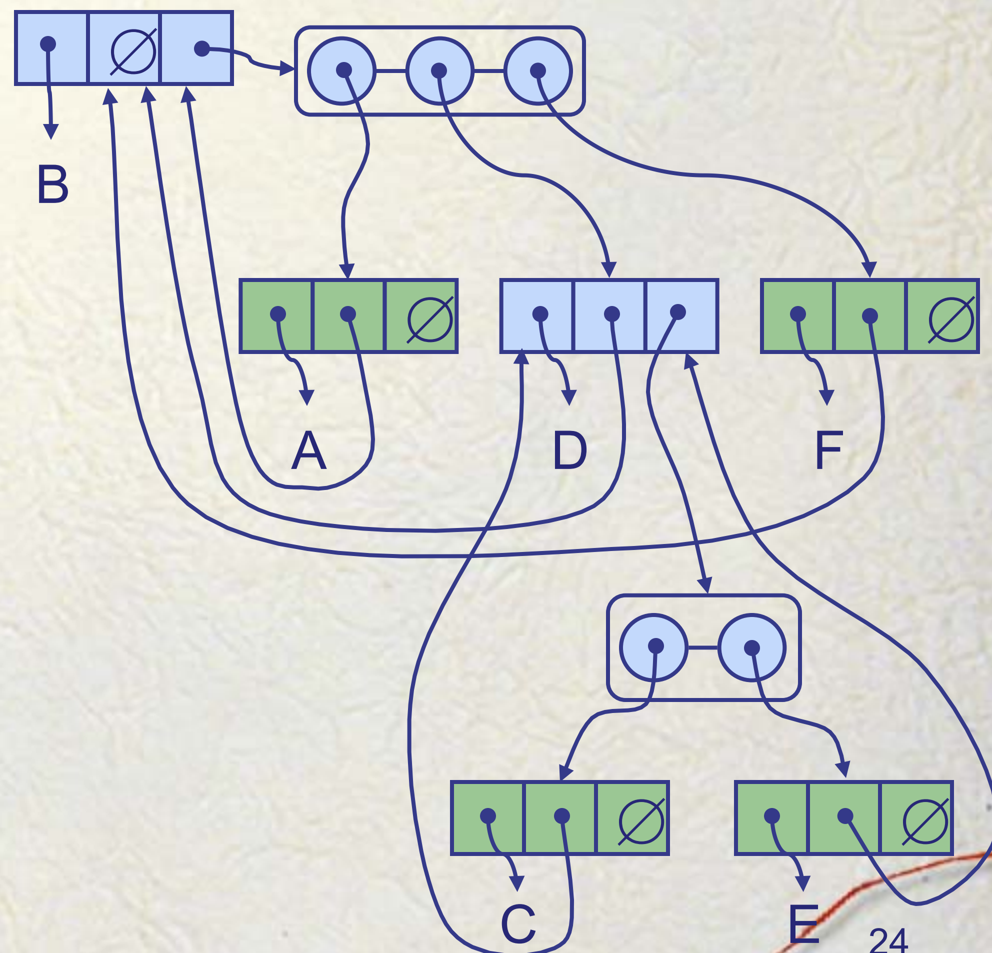
- We show how to specialize class EulerTour to evaluate an arithmetic expression
- Assumptions
  - External nodes store Integer objects
  - Internal nodes store Operator objects supporting method operation (Integer, Integer)

```
public class EvaluateExpression
                        extends EulerTour {

    protected void visitExternal(Position p, Result r) {
        r.finalResult = (Integer) p.element();
    }

    protected void visitRight(Position p, Result r) {
        Operator op = (Operator) p.element();
        r.finalResult = op.operation(
                            (Integer) r.leftResult,
                            (Integer) r.rightResult
                            );
    }

    …

}
```

Linked Structure for Trees

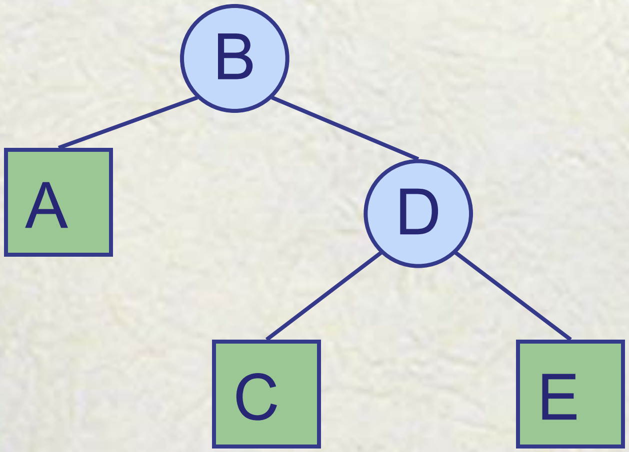
- A node is represented by an object storing
  - Element
  - Parent node
  - Sequence of children nodes
- Node objects implement the Position ADT

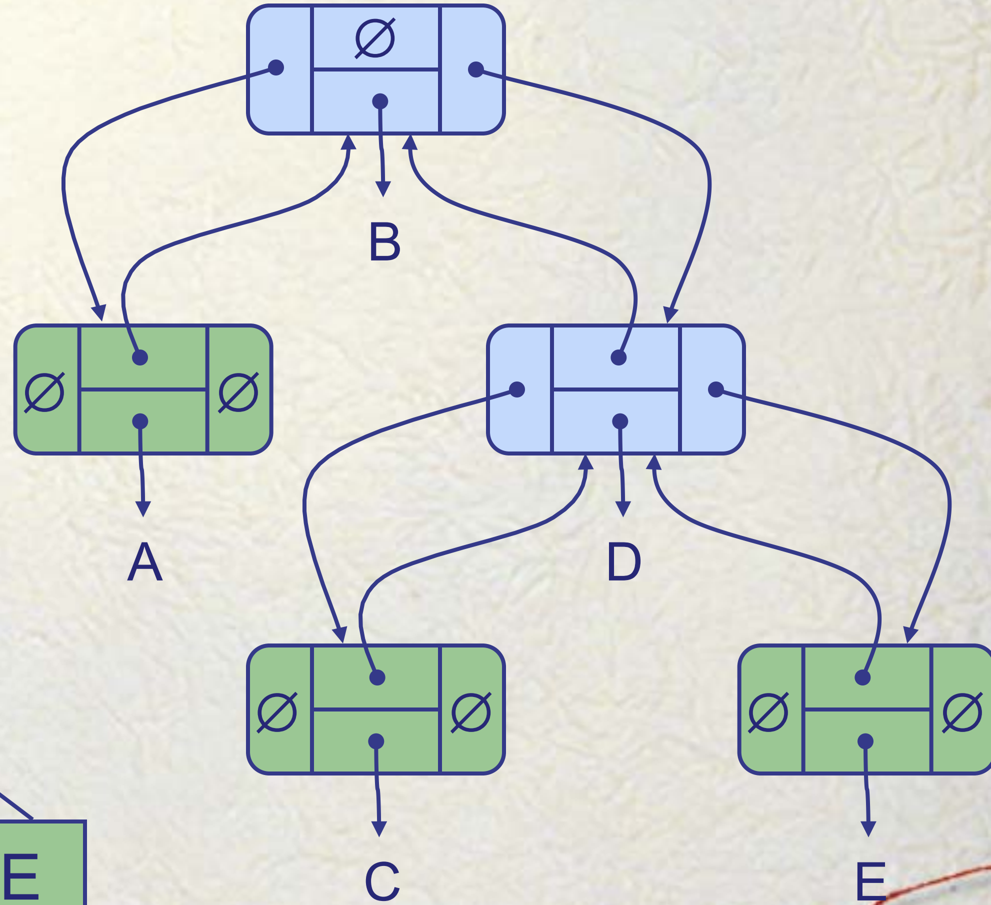




Linked Structure for Binary Trees

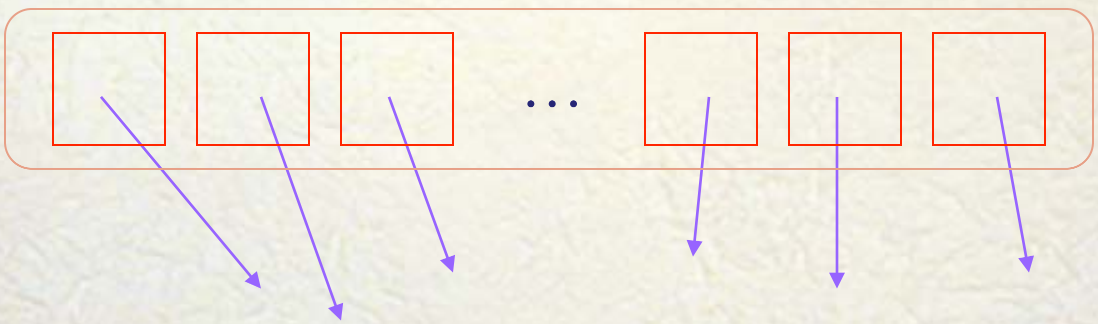
- A node is represented by an object storing
  - Element
  - Parent node
  - Left child node
  - Right child node
- Node objects implement the Position ADT

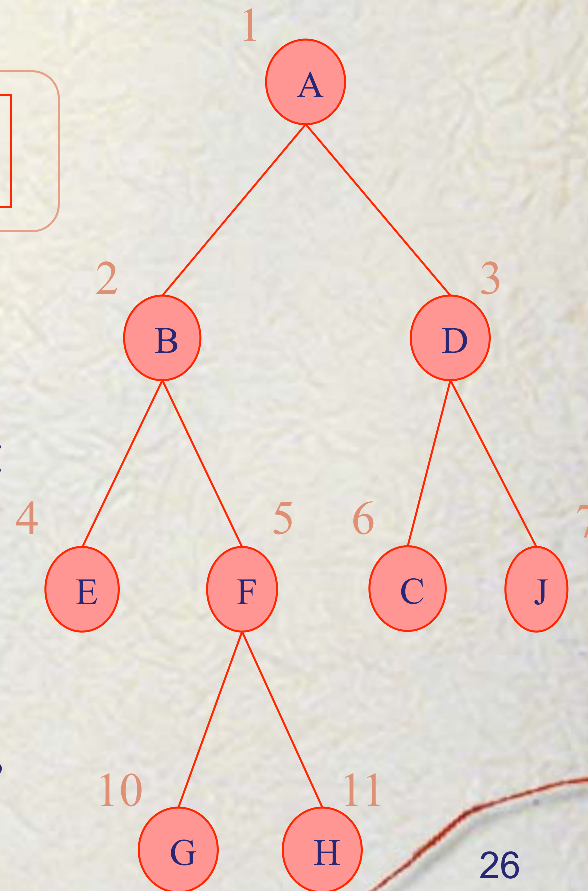




Array-Based Representation of Binary Trees

- nodes are stored in an array





- let rank(node) be defined as follows:
  - rank(root) = 1
  - if node is the left child of parent(node), rank(node) = 2*rank(parent(node))
  - if node is the right child of parent(node), rank(node) = 2*rank(parent(node))+1

References

- Chapter 7: Data structures and Algorithms by Goodrich and Tamassia.