Xét trên 1 đoạn [a. b]

0<$m_n \leq |f^{(n)}(x)| \leq M_n < +vô\ cùng$

Sai số

1. Chương 2:

- Chia đôi:

Dừng ở lần thứ n ta có:

a_n<nghiệm<b_n và b_n-a_n=$\frac{1}{2^n}$(b-a)

Lấy a_n là nghiệm, sai số là:

$|a_n - nghiệm| \leq b_n - a_n$=$\frac{1}{2^n}$(b-a)

Lấy b_n là nghiệm, sai số là:

$|b_n - nghiệm| \leq b_n - a_n$=$\frac{1}{2^n}$(b-a)

Lấy $\frac{a_n+b_n}{2}$ là nghiệm, sai số là:

$$\left|\frac{a_n + b_n}{2} - nghiệm\right| \leq \frac{b_n - a_n}{2} = \frac{1}{2^{n+1}}(b-a)$$

- Điểm bất động:

$|x_n - ξ| \leq \frac{M1}{1-M_1}|x_{n-1} - x_n|$

$|x_n - ξ| \leq \frac{{M_1}^n}{1-M_1}|x_1 - x_0|$

- Newton:

$$|x_n - ξ| \leq \frac{M_2}{2 * m_1}|x_{n-}x_{n-1}|^2$$

- Dây cung:

$$|x_n - \xi| \leq \frac{M_1 - m_1}{2 * m_1}|x_n - x_{n-1}|$$

- Điểm sai:

Ko thấy nói, chậm hơn dây cung.

2. Chương 3:

- Jacobi:

$$\left\|x^{(k)} - x^*\right\|_p \leq \frac{\|a\|_p}{1 - \|a\|_p}.\left\|x^{(k)} - x^{(k-1)}\right\|_p$$

$$\left\|x^{(k)} - x^*\right\|_p \leq \frac{(\|a\|_p)^k}{1 - \|a\|_p}.\left\|x^{(1)} - x^{(0)}\right\|_p$$

- Gauss-seidel:

Chuẩn hàng ∞:

$p_i = \sum_{j=1}^{i-1}\left|a_{ij}\right|$:hàng i ma trận tam giác dưới đường chéo chính

$q_i = \sum_{j=i}^{n}\left|a_{ij}\right|$:hàng I ma trận tam giác trên và bao gồm đường chéo chính

$\mu = \frac{max}{i}\frac{q_i}{1-p_i}$ i=1,n

$$\left\|x^{(k)} - x^*\right\|_\infty \leq \frac{\mu}{1 - \mu}.\left\|x^{(k)} - x^{(k-1)}\right\|_\infty$$

$$\left\|x^{(k)} - x^*\right\|_\infty \leq \frac{(\mu)^k}{1 - \mu}.\left\|x^{(1)} - x^{(0)}\right\|_\infty$$

Chuẩn cột 1:

$s = \frac{max}{j}\sum_{i=j+1}^{n}\left|a_{ij}\right|; \rho = \frac{max}{j}\frac{t_j}{1-s_j}$,(j=1,n)

$t_j = \sum_1^j\left|a_{ij}\right|, s_j = \sum_{i=j+1}^{n}\left|a_{ij}\right|$,(j=1,n-1)

$s_n = 0, t_n = \sum_1^n|a_{in}|$

$$\left\|x^{(k)} - x^*\right\|_1 \leq \frac{\rho}{(1-\mathrm{s}).(1-\rho)}.\left\|x^{(k)} - x^{(k-1)}\right\|_1$$

$$\left\|x^{(k)} - x^*\right\|_1 \leq \frac{(\rho)^k}{(1-\mathrm{s}).(1-\rho)}.\left\|x^{(1)} - x^{(0)}\right\|_1$$

3. Chương 4:

- Lagrange:

$$R_n(x) = \frac{f^{n+1}(c)}{(n+1)!}\pi_{n+1}(x)$$

$$|R_n(x)| \leq \frac{M_{n+1}}{(n+1)!}|\pi_{n+1}(x)|$$

$$\pi_{n+1}(x) = (x - x_0)(x - x_1)\ldots(x - x_n)$$

- Newton:

Không cách đều sai phân tiến:

$$R_n(x) = (x - x_0)(x - x_1)\ldots(x - x_{n-1})(x - x_n)f[x, x_0, \ldots, x_n]$$

$$R_n(x) = \pi_{n+1}(x)f[x, x_0, \ldots, x_n]$$

Không cách đều sai phân lùi:

$$R_n(x) = (x - x_n)(x - x_{n-1})\ldots(x - x_1)(x - x_0)f[x, x_n, \ldots, x_0]$$

$$R_n(x) = \pi_{n+1}(x)f[x, x_n, \ldots, x_0]$$

Cách đều hiệu hữu hạn tiến:

x=$x_0 + ht$

$$R_n(x) = \frac{h^{n+1}f^{(n+1)}(c)}{(n+1)!}t(t-1)..(t-n) \approx \frac{t(t-1)\ldots(t-n)}{(n+1)!}\Delta^{n+1}y_0$$

Cách đều hiệu hữu hạn lùi:

x=$x_n + ht$

$$R_n(x) = \frac{h^{n+1} f^{(n+1)}(c)}{(n+1)!} t(t+1)..(t+n) \approx \frac{t(t+1) \dots (t+n)}{(n+1)!} \nabla^{n+1} y_n$$

- Spline:

$$|f(x) - S(x)| \le \frac{5M}{384} \frac{max}{0 \le j \le n-1} (x_{j+1} - x_j)^4$$

$$max_{a \le x \le b} \left| f^{(4)}(x) \right| = M$$

Chương 5:

- Đạo hàm:

Đạo hàm lagrange 2 điểm: $\frac{h}{2} f''(\xi)$

Đạo hàm lagrange 3 điểm: $\frac{h^2}{3} f^{(3)}(\xi)$

Đạo hàm lagrange 3 điểm: $\frac{h^4}{5} f^{(5)}(\xi)$

$$f'(x_j) = \sum_{k=0}^{n} f(x_k) L'_k(x_j) + \frac{f^{(n+1)}(\xi(x_j))}{(n+1)!} \prod_{\substack{k=0 \\ k \ne j}}^{n} (x_j - x_k)$$

- Tích phân:

Tích phân hình chữ nhật:$\frac{h^2(b-a)}{24} . f''(\xi)$

Tích phân hình thang: $\frac{h^2(b-a)}{12} . f''(\xi)$

Tích phân simpson 3 điểm $\frac{h^4(b-a)}{180} f^{(4)}(\xi)$

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4. Chương 6:

Rời rạc: spare error từng điểm.

Đa thức bậc n (hilbert matrix): Tích phân sqare error trên đoạn [a, b]

Tập trực giao: Tương tự

5. Chương 7:

- Euler:

Lý thuyết: $|y_i - y(x_{i)}| \leq M.h$

- Euler cải tiến:

Lý thuyết: $|y_i - y(x_{i)}| \leq M.h^2$

Thực tế:

$$\left|y_{2n}\left(\frac{h}{2}\right) - y(X)\right| \approx \left|y_{2n}\left(\frac{h}{2}\right) - y_n(h)\right|$$

- Runge-Kutta 4 số hạng:

Lý thuyết: $|y_i - y(x_{i)}| \leq M.h^4$

(M=hằng số dương không phụ thuộc vào h)

Thực tế:

$$\left|y_{2n}\left(\frac{h}{2}\right) - y(X)\right| \approx \frac{1}{15}\left|y_{2n}\left(\frac{h}{2}\right) - y_n(h)\right|$$

Lý thuyết:

Chương 4:

$$g_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i$$

$$d_i = y_i$$

$$h_i = x_{i+1} - x_i$$

$$f_i = y_{i+1} - y_i$$

$$a_i = \frac{1}{3h_i}(b_{i+1} - b_i)$$

$$c_i = \frac{f_i}{h_i} - \frac{h_i}{3}(b_{i+1} + 2b_i)$$

$$\frac{1}{3}h_i b_i + \frac{2}{3}(h_i + h_{i+1})b_{i+1} + \frac{1}{3}h_{i+1}b_{i+2} = \frac{f_{i+1}}{h_{i+1}} - \frac{f_i}{h_i}$$

$$\begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \text{missing} & \cdots & \cdots \\ \frac{1}{3}h_0 & \frac{2}{3}(h_0+h_1) & \frac{1}{3}h_1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{3}h_{n-2} & \frac{2}{3}(h_{n-2}+h_{n-1}) & \frac{1}{3}h_{n-1} \\ \cdots & \cdots & \cdots & \cdots & \text{missing} & \cdots & \cdots \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \\ b_n \end{pmatrix} = \begin{pmatrix} \text{missing} \\ \frac{f_1}{h_1} - \frac{f_0}{h_0} \\ \vdots \\ \frac{f_{n-1}}{h_{n-1}} - \frac{f_{n-2}}{h_{n-2}} \\ \text{missing} \end{pmatrix}.$$

Clamped Cubin Spline

$g_0'(x) = \alpha, g_{n-1}'(x_n) = \beta$

First mising equation:

$$\frac{2}{3}h_0 b_0 + \frac{1}{3}h_0 b_1 = \frac{f_0}{h_0} - \alpha$$

Last missing equation:

$$\frac{1}{3}h_{n-1}b_{n-1} + \frac{2}{3}h_{n-1}b_n = \beta - \frac{f_{n-1}}{h_{n-1}}$$

Nature Cubin Spline:

$$\frac{1}{3}h_0b_0 = 0$$

$$\frac{1}{3}h_{n-1}b_n = 0$$

Hoặc:

$$h_1b_0 - (h_0 + h_1)b_1 + h_0b_2 = 0$$

$$h_{n-1}b_{n-2} - (h_{n-2} + h_{n-1})b_{n-1} + h_{n-2}b_n = 0$$

Chương 5:

Five-point Midpoint fomula:

$$f'(x_0) = \frac{1}{12h}[f(x_0 - 2h) - 8f(x_0 - h) + 8f(x_0 + h) - f(x_0 + 2h)] + \frac{h^4}{30}f^{(5)}(\xi),$$

Five-point Endpoint fomula:

$$f'(x_0) = \frac{1}{12h}[-25f(x_0) + 48f(x_0 + h) - 36f(x_0 + 2h) + 16f(x_0 + 3h) - 3f(x_0 + 4h)] + \frac{h^4}{5}f^{(5)}(\xi),$$

Second Derivative Midpoint Formula

$$f''(x_0) = \frac{1}{h^2}[f(x_0 - h) - 2f(x_0) + f(x_0 + h)] - \frac{h^2}{12}f^{(4)}(\xi),$$

Chương 6:

Xấp xỉ dữ liệu:

Đường thẳng:

$$a_0 = \frac{\sum_{i=1}^{m} x_i^2 \sum_{i=1}^{m} y_i - \sum_{i=1}^{m} x_i y_i \sum_{i=1}^{m} x_i}{m\left(\sum_{i=1}^{m} x_i^2\right) - \left(\sum_{i=1}^{m} x_i\right)^2}$$

$$a_1 = \frac{m\sum_{i=1}^{m} x_i y_i - \sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i}{m\left(\sum_{i=1}^{m} x_i^2\right) - \left(\sum_{i=1}^{m} x_i\right)^2}$$

Đa thức bậc n:

$$\begin{aligned}
a_0 \sum_{i=1}^{m} x_i^0 + a_1 \sum_{i=1}^{m} x_i^1 + a_2 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^n &= \sum_{i=1}^{m} y_i x_i^0 \\
a_0 \sum_{i=1}^{m} x_i^1 + a_1 \sum_{i=1}^{m} x_i^2 + a_2 \sum_{i=1}^{m} x_i^3 + \cdots + a_n \sum_{i=1}^{m} x_i^{n+1} &= \sum_{i=1}^{m} y_i x_i^1 \\
&\vdots \\
a_0 \sum_{i=1}^{m} x_i^n + a_1 \sum_{i=1}^{m} x_i^{n+1} + a_2 \sum_{i=1}^{m} x_i^{n+2} + \cdots + a_n \sum_{i=1}^{m} x_i^{2n} &= \sum_{i=1}^{m} y_i x_i^n
\end{aligned}$$

Hilbert matrix: Dùng tích phân thay vì sigma:

Solution The normal equations for $P_2(x) = a_2x^2 + a_1x + a_0$ are

$$a_0 \int_0^1 1\, dx + a_1 \int_0^1 x\, dx + a_2 \int_0^1 x^2\, dx = \int_0^1 \sin \pi x\, dx,$$

$$a_0 \int_0^1 x\, dx + a_1 \int_0^1 x^2\, dx + a_2 \int_0^1 x^3\, dx = \int_0^1 x \sin \pi x\, dx,$$

$$a_0 \int_0^1 x^2\, dx + a_1 \int_0^1 x^3\, dx + a_2 \int_0^1 x^4\, dx = \int_0^1 x^2 \sin \pi x\, dx.$$

Trực giao:

$$P(x) = \sum_{j=0}^{n} a_j \varphi_j(x)$$

$$a_j = \frac{\int_a^b w(x)\varphi_j(x) f(x) dx}{\int_a^b w(x)[\varphi_j]^2\, dx}$$

$\varphi_0 \equiv 1, \varphi_1 = x - B_1$

$$B_1 = \frac{\int_a^b x w(x)[\phi_0(x)]^2\, dx}{\int_a^b w(x)[\phi_0(x)]^2\, dx},$$

$$\phi_k(x) = (x - B_k)\phi_{k-1}(x) - C_k \phi_{k-2}(x),$$

$$B_k = \frac{\int_a^b x w(x)[\phi_{k-1}(x)]^2\, dx}{\int_a^b w(x)[\phi_{k-1}(x)]^2\, dx}$$

$$C_k = \frac{\int_a^b x w(x)\phi_{k-1}(x)\phi_{k-2}(x)\, dx}{\int_a^b w(x)[\phi_{k-2}(x)]^2\, dx}$$